

Upper and Lower Bounds Worksheet

Evaluating Formulae with Limits of Accuracy

I Do

The acceleration a m/s² of an object is calculated using the formula: $a = \frac{2(s-ut)}{t^2}$ where $s = 150$, $u = 12$, and $t = 4.0$ (all given correct to 1 decimal place).

By considering bounds, work out the upper bound for the value of a .

Show your working clearly.

We Do

The force F is calculated using the formula: $F = \frac{m(v-u)}{t}$

where $m = 5.4$, $v = 28.2$, $u = 14.6$, and $t = 3.5$ (all given correct to 1 decimal place).

By considering bounds, work out the upper bound for the value of F .

You Do

The power P is calculated using the formula: $P = \frac{E^2}{R}$

where $E = 12.5$ and $R = 4.2$ (both given correct to 1 decimal place).

By considering bounds, work out the upper bound for the value of P .

Challenge

y is calculated using the formula: $y = \frac{p-q}{r}$

$p = 45$ (correct to 2 significant figures), $q = 12$ (correct to 2 significant figures),

and $r = 8.5$ (correct to 1 decimal place).

Calculate the upper bound of y . Show your working clearly.

Answers & Worked Solutions

• I Do:

Values to 1 d.p. implies 150.0 ± 0.05 , 12.0 ± 0.05 , 4.0 ± 0.05 .

Upper bound of a : maximize numerator and minimize denominator.

$$a_{UB} = \frac{2(s_{UB} - u_{LB} \times t_{LB})}{(t_{LB})^2}$$

$$a_{UB} = \frac{2(150.05 - 11.95 \times 3.95)}{3.95^2} = \frac{2(150.05 - 47.2025)}{15.6025}$$

$$a_{UB} = \frac{2(102.8475)}{15.6025} = \frac{205.695}{15.6025} \approx 13.183 \quad \text{Answer: } 13.2 \text{ m/s}^2 \text{ (to 3 s.f.)}$$

• We Do:

Bounds: $m_{UB} = 5.45$, $v_{UB} = 28.25$, $u_{LB} = 14.55$, $t_{LB} = 3.45$.

$$F_{UB} = \frac{m_{UB}(v_{UB} - u_{LB})}{t_{LB}}$$

$$F_{UB} = \frac{5.45(28.25 - 14.55)}{3.45} = \frac{5.45(13.7)}{3.45}$$

$$F_{UB} = \frac{74.665}{3.45} \approx 21.642 \quad \text{Answer: } 21.6 \text{ (to 3 s.f.)}$$

• You Do:

Bounds: $E_{UB} = 12.55$, $R_{LB} = 4.15$.

$$P_{UB} = \frac{(E_{UB})^2}{R_{LB}}$$

$$P_{UB} = \frac{12.55^2}{4.15} = \frac{157.5025}{4.15} \approx 37.952 \quad \text{Answer: } 38.0 \text{ (to 3 s.f.)}$$

• Challenge:

Bounds: p is 45 to 2 s.f. (nearest 1) $\implies p_{UB} = 45.5$.

q is 12 to 2 s.f. (nearest 1) $\implies q_{LB} = 11.5$.

r is 8.5 to 1 d.p. $\implies r_{LB} = 8.45$.

$$y_{UB} = \frac{p_{UB} - q_{LB}}{r_{LB}} = \frac{45.5 - 11.5}{8.45} = \frac{34}{8.45} \approx 4.024 \quad \text{Answer: 4.02 (to 3 s.f.)}$$