

Probability & Algebra Worksheet

Forming and Solving Quadratics from Probability

I Do

A box contains n counters. 8 of the counters are red and the rest are blue.

Two counters are taken at random from the box without replacement.

Given that the probability that both counters are the same colour is $\frac{7}{15}$:

(a) Show that $n^2 - 31n + 240 = 0$.

(b) Solve $n^2 - 31n + 240 = 0$ to find the value of n .

We Do

A bag contains n sweets. 6 of the sweets are green and the rest are yellow.

Two sweets are taken at random from the bag without replacement.

The probability that both sweets are the same colour is $\frac{1}{2}$.

(a) Show that $n^2 - 25n + 144 = 0$.

(b) Find the value of n .

You Do

A box contains n marbles. 5 of the marbles are white and the rest are black.

Two marbles are taken at random from the box without replacement.

The probability that both marbles are the same colour is $\frac{5}{9}$.

(a) Show that $2n^2 - 47n + 225 = 0$.

(b) Find the value of n .

Challenge

A jar contains 10 counters. x of the counters are red and the rest are blue.

Two counters are taken at random.

The probability that they are both the same colour is $\frac{7}{15}$.

Find the possible values of x .

Answers & Worked Solutions

• I Do:

$$(a) P(RR) = \frac{8}{n} \times \frac{7}{n-1} = \frac{56}{n(n-1)}, P(BB) = \frac{n-8}{n} \times \frac{n-9}{n-1} = \frac{n^2-17n+72}{n(n-1)}.$$

$$P(\text{Same}) = \frac{n^2-17n+128}{n(n-1)} = \frac{7}{15} \implies 15(n^2-17n+128) = 7(n^2-n)$$

$$15n^2 - 255n + 1920 = 7n^2 - 7n \implies 8n^2 - 248n + 1920 = 0$$

$$\text{Divide by 8: } n^2 - 31n + 240 = 0 \quad \blacksquare$$

$$(b) (n-15)(n-16) = 0 \implies n = 15 \text{ or } n = 16. \quad \text{Answer: 15 or 16}$$

• We Do:

$$(a) P(GG) = \frac{30}{n(n-1)}, P(YY) = \frac{(n-6)(n-7)}{n(n-1)} = \frac{n^2-13n+42}{n(n-1)}$$

$$P(\text{Same}) = \frac{n^2-13n+72}{n^2-n} = \frac{1}{2} \implies 2n^2 - 26n + 144 = n^2 - n$$

$$n^2 - 25n + 144 = 0 \quad \blacksquare$$

$$(b) (n-9)(n-16) = 0 \implies n = 9 \text{ or } 16. \quad \text{Answer: 9 or 16}$$

• You Do:

$$(a) P(WW) = \frac{20}{n(n-1)}, P(BB) = \frac{(n-5)(n-6)}{n(n-1)} = \frac{n^2-11n+30}{n(n-1)}$$

$$P(\text{Same}) = \frac{n^2-11n+50}{n^2-n} = \frac{5}{9} \implies 9n^2 - 99n + 450 = 5n^2 - 5n$$

$$4n^2 - 94n + 450 = 0 \implies 2n^2 - 47n + 225 = 0 \quad \blacksquare$$

$$(b) (2n - 25)(n - 9) = 0 \implies n = 12.5 \text{ (reject) or } n = 9. \quad \text{Answer: 9}$$

• Challenge:

$$P(RR) = \frac{x(x-1)}{90}, P(BB) = \frac{(10-x)(9-x)}{90}$$

$$P(\text{Same}) = \frac{x^2-x+90-19x+x^2}{90} = \frac{7}{15} = \frac{42}{90}$$

$$2x^2 - 20x + 90 = 42 \implies 2x^2 - 20x + 48 = 0 \implies x^2 - 10x + 24 = 0$$

$$(x - 4)(x - 6) = 0 \implies x = 4 \text{ or } 6. \quad \text{Answer: 4 or 6}$$