

Functions Worksheet

Inverse and Composite Functions

I Do

The functions f and g are defined as: $f(x) = \frac{2x+1}{x-3}$, $x \neq 3$ and $g(x) = x^2 - 4$.

- Find $f^{-1}(x)$.
- Express $gf(x)$ as a single fraction in its simplest form.
- Solve $g(x) = 21$.

We Do

The functions f and g are defined as: $f(x) = \frac{3x+2}{x-1}$, $x \neq 1$ and $g(x) = x^2 - 1$.

- Find $f^{-1}(x)$.
- Express $gf(x)$ as a single fraction in its simplest form.
- Solve $g(x) = 8$.

You Do

The functions f and g are defined as: $f(x) = \frac{x+4}{2x-1}$, $x \neq 0.5$ and $g(x) = x^2 + 3$.

- Find $f^{-1}(x)$.
- Express $gf(x)$ as a single fraction in its simplest form.
- Solve $g(x) = 19$.

Challenge

The function h is defined as $h(x) = \frac{ax+b}{cx+d}$.

Show algebraically that if $a = -d$, then $h^{-1}(x) = h(x)$.

Answers & Worked Solutions

• I Do:

$$(a) y = \frac{2x+1}{x-3} \implies y(x-3) = 2x+1 \implies yx-3y = 2x+1 \implies x(y-2) = 3y+1$$

$$x = \frac{3y+1}{y-2} \quad \text{Answer: } f^{-1}(x) = \frac{3x+1}{x-2}$$

$$(b) gf(x) = \left(\frac{2x+1}{x-3}\right)^2 - 4 = \frac{4x^2+4x+1}{(x-3)^2} - \frac{4(x^2-6x+9)}{(x-3)^2} = \frac{28x-35}{(x-3)^2}$$

$$(c) x^2 - 4 = 21 \implies x^2 = 25 \implies x = \pm 5$$

• We Do:

$$(a) y(x-1) = 3x+2 \implies xy-y = 3x+2 \implies x(y-3) = y+2 \quad \text{Answer: } f^{-1}(x) = \frac{x+2}{x-3}$$

$$(b) gf(x) = \left(\frac{3x+2}{x-1}\right)^2 - 1 = \frac{9x^2+12x+4-1(x^2-2x+1)}{(x-1)^2} = \frac{8x^2+14x+3}{(x-1)^2}$$

$$(c) x^2 - 1 = 8 \implies x^2 = 9 \implies x = \pm 3$$

• You Do:

$$(a) y(2x-1) = x+4 \implies 2xy-y = x+4 \implies x(2y-1) = y+4 \quad \text{Answer: } f^{-1}(x) = \frac{x+4}{2x-1}$$

$$(b) \quad gf(x) = \left(\frac{x+4}{2x-1}\right)^2 + 3 = \frac{x^2+8x+16+3(4x^2-4x+1)}{(2x-1)^2} = \frac{13x^2-4x+19}{(2x-1)^2}$$

$$(c) \quad x^2 + 3 = 19 \implies x^2 = 16 \implies x = \pm 4$$

• **Challenge:**

$$\text{Let } y = \frac{ax+b}{cx+d} \implies y(cx+d) = ax+b \implies cyx + dy = ax+b \implies x(cy-a) = b-dy$$

$$x = \frac{-dy+b}{cy-a} \implies h^{-1}(x) = \frac{-dx+b}{cx-a}$$

If $a = -d$, then substitute $-d = a$ into numerator and $-a = d$ into denominator:

$$h^{-1}(x) = \frac{ax+b}{cx+d} = h(x) \quad \blacksquare$$