

Calculus & Optimisation Worksheet

Finding Maximum Volume using Differentiation

I Do

An open-topped storage tank has a square base of length x m and vertical height h m. The total external surface area of the tank is 108 m^2 .

(a) Show that the volume $V \text{ m}^3$ of the tank is given by $V = 27x - \frac{1}{4}x^3$.

(b) Use calculus $\left(\frac{dV}{dx}\right)$ to find the maximum volume of the tank. Show clear algebraic working.

We Do

An open-topped storage tank has a square base of length x m and vertical height h m. The total external surface area of the tank is 192 m^2 .

(a) Show that the volume $V \text{ m}^3$ of the tank is given by $V = 48x - \frac{1}{4}x^3$.

(b) Use calculus to find the maximum volume of the tank.

You Do

An open-topped box has a square base of length x cm and vertical height h cm. The total surface area of the box is 300 cm^2 .

(a) Show that the volume $V \text{ cm}^3$ of the box is given by $V = 75x - \frac{1}{4}x^3$.

(b) Use calculus to find the maximum volume of the box.

Challenge

A closed solid cylinder has base radius r cm and height h cm. The total surface area of the cylinder is $600\pi \text{ cm}^2$.

(a) Show that the volume $V \text{ cm}^3$ is given by $V = 300\pi r - \pi r^3$.

(b) Find the maximum volume of the cylinder in terms of π .

Answers & Worked Solutions

• I Do:

$$(a) \text{ Surface Area} = x^2 + 4xh = 108 \implies 4xh = 108 - x^2 \implies h = \frac{108 - x^2}{4x}.$$

$$\text{Volume } V = x^2h = x^2 \left(\frac{108 - x^2}{4x} \right) = \frac{108x^2 - x^4}{4x} = 27x - \frac{1}{4}x^3 \quad \blacksquare$$

$$(b) \frac{dV}{dx} = 27 - \frac{3}{4}x^2 = 0 \implies \frac{3}{4}x^2 = 27 \implies x^2 = 36 \implies x = 6.$$

$$V_{\max} = 27(6) - \frac{1}{4}(6)^3 = 162 - \frac{1}{4}(216) = 162 - 54 = 108. \quad \text{Answer: } 108 \text{ m}^3$$

• We Do:

$$(a) SA = x^2 + 4xh = 192 \implies h = \frac{192 - x^2}{4x}.$$

$$V = x^2h = x^2 \left(\frac{192 - x^2}{4x} \right) = \frac{192x^2 - x^4}{4x} = 48x - \frac{1}{4}x^3 \quad \blacksquare$$

$$(b) \frac{dV}{dx} = 48 - \frac{3}{4}x^2 = 0 \implies \frac{3}{4}x^2 = 48 \implies x^2 = 64 \implies x = 8.$$

$$V_{\max} = 48(8) - \frac{1}{4}(8)^3 = 384 - 128 = 256. \quad \text{Answer: } 256 \text{ m}^3$$

• You Do:

$$(a) SA = x^2 + 4xh = 300 \implies h = \frac{300 - x^2}{4x}.$$

$$V = x^2 \left(\frac{300-x^2}{4x} \right) = 75x - \frac{1}{4}x^3 \quad \blacksquare$$

$$(b) \frac{dV}{dx} = 75 - \frac{3}{4}x^2 = 0 \implies x^2 = 100 \implies x = 10.$$

$$V_{max} = 75(10) - \frac{1}{4}(10)^3 = 750 - 250 = 500. \quad \text{Answer: } 500 \text{ cm}^3$$

• **Challenge:**

$$(a) \text{ Total SA} = 2\pi r^2 + 2\pi rh = 600\pi \implies r^2 + rh = 300 \implies h = \frac{300-r^2}{r}.$$

$$V = \pi r^2 h = \pi r^2 \left(\frac{300-r^2}{r} \right) = \pi r(300 - r^2) = 300\pi r - \pi r^3 \quad \blacksquare$$

$$(b) \frac{dV}{dr} = 300\pi - 3\pi r^2 = 0 \implies 3\pi r^2 = 300\pi \implies r^2 = 100 \implies r = 10.$$

$$V_{max} = 300\pi(10) - \pi(10)^3 = 3000\pi - 1000\pi = 2000\pi. \quad \text{Answer: } 2000\pi \text{ cm}^3$$