

Non-Linear Simultaneous Equations Worksheet

Intersecting Lines and Curves

I Do

The straight line L has equation $2x + y = 7$. The curve C has equation $x^2 + y^2 - 4x + 2y = 32$. Find the coordinates of the points where L and C intersect. Show clear algebraic working.

We Do

The straight line L has equation $x - y = 2$. The curve C has equation $x^2 + y^2 + 6x - 4y = 12$. Find the coordinates of the points where L and C intersect. Show clear algebraic working.

You Do

The straight line L has equation $2x - y = -2$. The curve C has equation $x^2 + y^2 - 4x = 5$. Find the coordinates of the points where L and C intersect. Show clear algebraic working.

Challenge

The straight line L has equation $3x + 4y = 12$. The curve C has equation $xy = -9$. Find the coordinates of the points where L and C intersect. Show clear algebraic working.

Answers & Worked Solutions

• I Do:

$y = 7 - 2x$. *Substitute into the circle equation:*

$$x^2 + (7 - 2x)^2 - 4x + 2(7 - 2x) = 32$$

$$x^2 + 49 - 28x + 4x^2 - 4x + 14 - 4x = 32$$

$$5x^2 - 36x + 63 = 32 \implies 5x^2 - 36x + 31 = 0$$

Factorise: $(5x - 31)(x - 1) = 0 \implies x = \frac{31}{5} = 6.2 \text{ or } x = 1$

When $x = 6.2$, $y = 7 - 2(6.2) = -5.4$. *When* $x = 1$, $y = 7 - 2(1) = 5$.

Answer: $(1, 5)$ and $(6.2, -5.4)$

• We Do:

$x = y + 2$. *Substitute into the curve equation:*

$$(y + 2)^2 + y^2 + 6(y + 2) - 4y = 12$$

$$y^2 + 4y + 4 + y^2 + 6y + 12 - 4y = 12$$

$$2y^2 + 6y + 16 = 12 \implies 2y^2 + 6y + 4 = 0 \implies y^2 + 3y + 2 = 0$$

Factorise: $(y + 1)(y + 2) = 0 \implies y = -1 \text{ or } y = -2$

When $y = -1$, $x = 1$. *When* $y = -2$, $x = 0$.

Answer: $(1, -1)$ and $(0, -2)$

• You Do:

$y = 2x + 2$. *Substitute into the curve equation:*

$$x^2 + (2x + 2)^2 - 4x = 5$$

$$x^2 + 4x^2 + 8x + 4 - 4x = 5$$

$$5x^2 + 4x - 1 = 0$$

Factorise: $(5x - 1)(x + 1) = 0 \implies x = 0.2 \text{ or } x = -1$

When $x = 0.2$, $y = 2(0.2) + 2 = 2.4$. **When** $x = -1$, $y = 2(-1) + 2 = 0$.

Answer: $(0.2, 2.4)$ and $(-1, 0)$

• **Challenge:**

From L, $4y = 12 - 3x \implies y = \frac{12-3x}{4}$. **Substitute into** $xy = -9$:

$$x \left(\frac{12-3x}{4} \right) = -9 \implies 12x - 3x^2 = -36$$

$$3x^2 - 12x - 36 = 0 \implies x^2 - 4x - 12 = 0$$

Factorise: $(x - 6)(x + 2) = 0 \implies x = 6 \text{ or } x = -2$

When $x = 6$, $y = -\frac{9}{6} = -1.5$. **When** $x = -2$, $y = \frac{-9}{-2} = 4.5$.

Answer: $(6, -1.5)$ and $(-2, 4.5)$