

Vectors Worksheet

Vector Geometry and Parallel Proofs

I Do

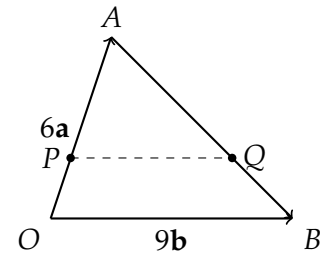
In triangle OAB , $\vec{OA} = 6\mathbf{a}$ and $\vec{OB} = 9\mathbf{b}$.

P is the point on OA such that $OP : PA = 1 : 2$.

Q is the point on AB such that $AQ : QB = 2 : 1$.

(a) Find $\vec{PQ}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(b) Prove that PQ is parallel to OB and state the ratio $PQ : OB$.



We Do

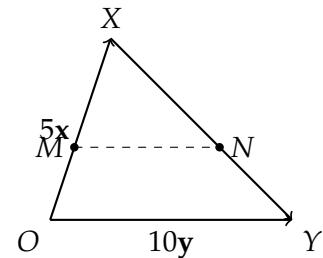
In triangle OXY , $\vec{OX} = 5\mathbf{x}$ and $\vec{OY} = 10\mathbf{y}$.

M is the point on OX such that $OM : MX = 2 : 3$.

N is the point on XY such that $XN : NY = 3 : 2$.

(a) Find $\vec{MN}$ in terms of $\mathbf{x}$ and $\mathbf{y}$.

(b) Prove that MN is parallel to OY and state the ratio $MN : OY$.



You Do

In triangle OEF , $\vec{OE} = 8\mathbf{e}$ and $\vec{OF} = 12\mathbf{f}$.

R is the point on OE such that $OR : RE = 1 : 3$.

S is the point on EF such that $ES : SF = 3 : 1$.

(a) Find $\vec{RS}$ in terms of $\mathbf{e}$ and $\mathbf{f}$.

(b) Prove that RS is parallel to OF and state the ratio $RS : OF$.

Challenge

$OABC$ is a parallelogram where $\vec{OA} = \mathbf{a}$ and $\vec{OC} = \mathbf{c}$.

X is the midpoint of the diagonal AC .

Y is the point on AB such that $AY : YB = 1 : 2$.

Find the vector $\vec{XY}$ in terms of $\mathbf{a}$ and $\mathbf{c}$ in its simplest form.

Answers & Worked Solutions

• I Do:

(a) $\vec{AB} = -6\mathbf{a} + 9\mathbf{b}$. $\vec{AQ} = \frac{2}{3}(-6\mathbf{a} + 9\mathbf{b}) = -4\mathbf{a} + 6\mathbf{b}$. $\vec{PA} = \frac{2}{3}(6\mathbf{a}) = 4\mathbf{a}$.

$\vec{PQ} = \vec{PA} + \vec{AQ} = 4\mathbf{a} + (-4\mathbf{a} + 6\mathbf{b}) = 6\mathbf{b}$. *Answer: $6\mathbf{b}$*

(b) $\vec{PQ} = 6\mathbf{b}$, $\vec{OB} = 9\mathbf{b}$. $\vec{PQ} = \frac{2}{3}\vec{OB}$, so they are scalar multiples of each other, hence parallel. Ratio is $6 : 9 = 2 : 3$.

• We Do:

(a) $\vec{XY} = -5\mathbf{x} + 10\mathbf{y}$. $\vec{XN} = \frac{3}{5}(-5\mathbf{x} + 10\mathbf{y}) = -3\mathbf{x} + 6\mathbf{y}$. $\vec{MX} = \frac{3}{5}(5\mathbf{x}) = 3\mathbf{x}$.

$\vec{MN} = \vec{MX} + \vec{XN} = 3\mathbf{x} + (-3\mathbf{x} + 6\mathbf{y}) = 6\mathbf{y}$. *Answer: $6\mathbf{y}$*

(b) $\vec{MN} = 6\mathbf{y}$, $\vec{OY} = 10\mathbf{y}$. $\vec{MN} = \frac{3}{5}\vec{OY} \implies$ parallel. Ratio is $6 : 10 = 3 : 5$.

• You Do:

(a) $\vec{EF} = -8\mathbf{e} + 12\mathbf{f}$. $\vec{ES} = \frac{3}{4}(-8\mathbf{e} + 12\mathbf{f}) = -6\mathbf{e} + 9\mathbf{f}$. $\vec{RE} = \frac{3}{4}(8\mathbf{e}) = 6\mathbf{e}$.

$$\vec{RS} = \vec{RE} + \vec{ES} = 6\mathbf{e} + (-6\mathbf{e} + 9\mathbf{f}) = 9\mathbf{f}. \quad \text{Answer: } 9\mathbf{f}$$

$$(b) \vec{RS} = 9\mathbf{f}, \vec{OF} = 12\mathbf{f}. \vec{RS} = \frac{3}{4}\vec{OF} \implies \text{parallel. Ratio is } 9 : 12 = 3 : 4.$$

• Challenge:

$$\vec{OB} = \mathbf{a} + \mathbf{c}. \vec{AC} = \mathbf{c} - \mathbf{a}. \vec{OX} = \vec{OA} + \frac{1}{2}\vec{AC} = \mathbf{a} + \frac{1}{2}(\mathbf{c} - \mathbf{a}) = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c}.$$

$$\vec{AB} = \mathbf{c}. \vec{AY} = \frac{1}{3}\vec{AB} = \frac{1}{3}\mathbf{c}. \vec{OY} = \vec{OA} + \vec{AY} = \mathbf{a} + \frac{1}{3}\mathbf{c}.$$

$$\vec{XY} = \vec{OY} - \vec{OX} = (\mathbf{a} + \frac{1}{3}\mathbf{c}) - (\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c}) = \frac{1}{2}\mathbf{a} - \frac{1}{6}\mathbf{c}. \quad \text{Answer: } \frac{1}{2}\mathbf{a} - \frac{1}{6}\mathbf{c}$$