

3D Trigonometry Worksheet

Pyramids, Face Angles, and Slant Heights

I Do

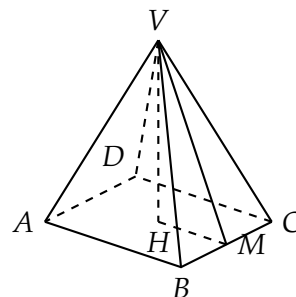
The diagram shows a right pyramid $VABCD$ with a horizontal square base $ABCD$ of side length 10 cm.

The vertical height VH is 15 cm, where H is the centre of the base.

M is the midpoint of BC .

(a) Calculate the length of VM . Give your answer correct to 3 significant figures.

(b) Calculate the angle between the triangular face VBC and the base plane $ABCD$. Give your answer correct to 1 decimal place.



We Do

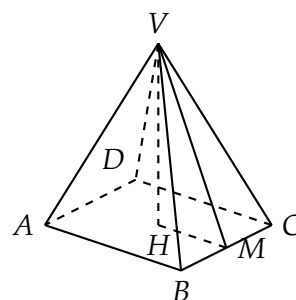
The diagram shows a right pyramid $VABCD$ with a horizontal square base $ABCD$ of side length 12 cm.

The vertical height VH is 8 cm, where H is the centre of the base.

M is the midpoint of BC .

(a) Calculate the exact length of VM .

(b) Calculate the angle between the triangular face VBC and the base plane $ABCD$.



You Do

A right pyramid $VABCD$ has a horizontal square base $ABCD$ of side length 16 cm.

The vertical height VH is 15 cm, where H is the centre of the base. M is the midpoint of BC .

(a) Calculate the exact length of VM .

(b) Calculate the angle between the triangular face VBC and the base plane $ABCD$.

Challenge

A right pyramid has a square base of side length $2x$. The angle between any triangular face and the base plane is 60° .

Show that the total surface area of the pyramid (including the base) is exactly $12x^2$.

Answers & Worked Solutions

• I Do:

(a) HM is half the base length = 5 cm. In $\triangle VHM$ (right-angled at H): $VM^2 = VH^2 + HM^2 = 15^2 + 5^2 = 250$.

$VM = \sqrt{250} \approx 15.811$ cm. Answer: 15.8 cm

(b) The required angle is $\angle VMH$. $\tan(\theta) = \frac{VH}{HM} = \frac{15}{5} = 3 \implies \theta = \tan^{-1}(3) \approx 71.565^\circ$. Answer: 71.6°

• We Do:

(a) $HM = 6$ cm. $VM^2 = 8^2 + 6^2 = 64 + 36 = 100 \implies VM = \sqrt{100} = 10$ cm. Answer: 10 cm

(b) $\tan(\angle VMH) = \frac{VH}{HM} = \frac{8}{6} = \frac{4}{3} \implies \angle VMH = \tan^{-1}\left(\frac{4}{3}\right) \approx 53.13^\circ$. Answer: 53.1°

• You Do:

(a) $HM = 8 \text{ cm}$. $VM^2 = 15^2 + 8^2 = 225 + 64 = 289 \implies VM = \sqrt{289} = 17 \text{ cm}$. *Answer: 17 cm*
 (b) $\tan(\angle VMH) = \frac{VH}{HM} = \frac{15}{8} \implies \angle VMH = \tan^{-1}\left(\frac{15}{8}\right) \approx 61.927^\circ$. *Answer: 61.9°*

• Challenge:

Distance from center to edge $HM = x$. *Angle* $\angle VMH = 60^\circ$.

Slant height VM : $\cos(60^\circ) = \frac{HM}{VM} \implies 0.5 = \frac{x}{VM} \implies VM = 2x$.

Area of base $= (2x)^2 = 4x^2$.

Area of one triangular face $= \frac{1}{2} \times \text{base} \times \text{slant height} = \frac{1}{2}(2x)(2x) = 2x^2$.

Total Area $= \text{Base} + 4 \times \text{Faces} = 4x^2 + 4(2x^2) = 4x^2 + 8x^2 = 12x^2$ ■