

Rationalising the Denominator Mastery

Practice Worksheet

I DO

Show that the following expression can be written in the form $a + b\sqrt{3}$, where a and b are integers:

$$\frac{(2 + \sqrt{3})^2}{2 - \sqrt{3}}$$

Show all stages of your working clearly.

WE DO

Show that the following expression can be written in the form $a + b\sqrt{7}$, where a and b are integers:

$$\frac{(3 + \sqrt{7})^2}{3 - \sqrt{7}}$$

Show all stages of your working clearly.

YOU DO 1

Show that the following expression can be written in the form $a + b\sqrt{15}$, where a and b are integers:

$$\frac{(4 + \sqrt{15})^2}{4 - \sqrt{15}}$$

Show all stages of your working clearly.

YOU DO 2

Show that the following expression can be written in the form $a + b\sqrt{21}$, where a and b are integers:

$$\frac{(5 - \sqrt{21})^2}{5 + \sqrt{21}}$$

Show all stages of your working clearly.

CHALLENGE

Show that the following expression can be written in the form $a + b\sqrt{5}$, where a and b are integers:

$$\frac{(2\sqrt{5} + 3)^2}{\sqrt{5} - 2}$$

Show all stages of your working clearly.

ANSWERS & WORKED SOLUTIONS

I Do

Step 1: Expand the numerator: $(2 + \sqrt{3})^2 = (2 + \sqrt{3})(2 + \sqrt{3}) = 4 + 2\sqrt{3} + 2\sqrt{3} + 3 = 7 + 4\sqrt{3}$.

Step 2: The expression is now $\frac{7 + 4\sqrt{3}}{2 - \sqrt{3}}$.

Step 3: Rationalise the denominator by multiplying the numerator and denominator by the conjugate $(2 + \sqrt{3})$.

Step 4: New numerator: $(7 + 4\sqrt{3})(2 + \sqrt{3}) = 14 + 7\sqrt{3} + 8\sqrt{3} + 12 = 26 + 15\sqrt{3}$.

Step 5: New denominator: $(2 - \sqrt{3})(2 + \sqrt{3}) = 4 + 2\sqrt{3} - 2\sqrt{3} - 3 = 4 - 3 = 1$.

Step 6: Write the final expression: $\frac{26 + 15\sqrt{3}}{1} = 26 + 15\sqrt{3}$. **Answer: $26 + 15\sqrt{3}$**

We Do

Step 1: Expand the numerator: $(3 + \sqrt{7})^2 = (3 + \sqrt{7})(3 + \sqrt{7}) = 9 + 3\sqrt{7} + 3\sqrt{7} + 7 = 16 + 6\sqrt{7}$.

Step 2: Multiply numerator and denominator by the conjugate $(3 + \sqrt{7})$.

Step 3: New numerator: $(16 + 6\sqrt{7})(3 + \sqrt{7}) = 48 + 16\sqrt{7} + 18\sqrt{7} + 42 = 90 + 34\sqrt{7}$.

Step 4: New denominator: $(3 - \sqrt{7})(3 + \sqrt{7}) = 9 - 7 = 2$.

Step 5: Divide by 2: $\frac{90 + 34\sqrt{7}}{2} = 45 + 17\sqrt{7}$. **Answer: $45 + 17\sqrt{7}$**

ANSWERS & WORKED SOLUTIONS (CONTINUED)

You Do 1

Step 1: Expand the numerator: $(4 + \sqrt{15})^2 = 16 + 4\sqrt{15} + 4\sqrt{15} + 15 = 31 + 8\sqrt{15}$.

Step 2: Multiply numerator and denominator by the conjugate $(4 + \sqrt{15})$.

Step 3: New numerator: $(31 + 8\sqrt{15})(4 + \sqrt{15}) = 124 + 31\sqrt{15} + 32\sqrt{15} + 120 = 244 + 63\sqrt{15}$.

Step 4: New denominator: $(4 - \sqrt{15})(4 + \sqrt{15}) = 16 - 15 = 1$.

Step 5: The final result is $244 + 63\sqrt{15}$. **Answer: $244 + 63\sqrt{15}$**

You Do 2

Step 1: Expand the numerator: $(5 - \sqrt{21})^2 = 25 - 5\sqrt{21} - 5\sqrt{21} + 21 = 46 - 10\sqrt{21}$.

Step 2: Multiply numerator and denominator by the conjugate $(5 + \sqrt{21})$.

Step 3: New numerator: $(46 - 10\sqrt{21})(5 + \sqrt{21}) = 230 - 46\sqrt{21} - 50\sqrt{21} + 210 = 440 - 96\sqrt{21}$.

Step 4: New denominator: $(5 + \sqrt{21})(5 - \sqrt{21}) = 25 - 21 = 4$.

Step 5: Divide by 4: $\frac{440 - 96\sqrt{21}}{4} = 110 - 24\sqrt{21}$. **Answer: $110 - 24\sqrt{21}$**

ANSWERS & WORKED SOLUTIONS (CONTINUED)

Challenge

Step 1: Expand the numerator: $(2\sqrt{5} + 3)^2 = (2\sqrt{5})(2\sqrt{5}) + 3(2\sqrt{5}) + 3(2\sqrt{5}) + 9 = 20 + 6\sqrt{5} + 6\sqrt{5} + 9 = 29 + 12\sqrt{5}$.

Step 2: Multiply numerator and denominator by the conjugate $(\sqrt{5} + 2)$.

Step 3: New numerator: $(29 + 12\sqrt{5})(\sqrt{5} + 2) = 29\sqrt{5} + 58 + 12(5) + 24\sqrt{5} = 29\sqrt{5} + 58 + 60 + 24\sqrt{5} = 118 + 53\sqrt{5}$.

Step 4: New denominator: $(\sqrt{5} - 2)(\sqrt{5} + 2) = 5 - 4 = 1$.

Step 5: The final result is $118 + 53\sqrt{5}$. **Answer: $118 + 53\sqrt{5}$**