ALGEBRA: ALGEBRAIC PROOF

Best Guess Mastery Worksheet (Target Grade 7/8)

I Do

Prove algebraically that the sum of the squares of any two consecutive odd integers is always 2 more than a multiple of 8.

We Do

Prove algebraically that the sum of any three consecutive integers is always a multiple of 3.

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You Do 1

Prove algebraically that the product of any two even numbers is always a multiple of 4.

You Do 2

Prove algebraically that the difference between the squares of any two consecutive integers is always equal to the sum of the two integers.

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Challenge 1

Prove algebraically that the sum of any four consecutive integers is **never** a multiple of 4.

Challenge 2

Prove algebraically that $(n + 3)^2 - n(n + 6)$ is a perfect square for all integer values of n.

Answers & Worked Solutions

I Do:

1. Let the two consecutive odd integers be $(2n + 1)$ and $(2n + 3)$.
2. Set up the sum of their squares: $(2n + 1)^2 + (2n + 3)^2$
3. Expand: $(4n^2 + 4n + 1) + (4n^2 + 12n + 9)$
4. Simplify: $8n^2 + 16n + 10$
5. Factor out 8 from the n terms: $8(n^2 + 2n + 1) + 2$
6. Conclusion: Since $8(n^2 + 2n + 1)$ is a multiple of 8, the expression is always **2 more than a multiple of 8**.

We Do:

1. Let the three consecutive integers be n, $(n + 1)$, and $(n + 2)$.
2. Set up the sum: $n + (n + 1) + (n + 2)$
3. Simplify: $3n + 3$
4. Factor out 3: $3(n + 1)$
5. Conclusion: Since 3 is a factor, 3 × (integer) is always a **multiple of 3**.

You Do 1:

1. Let the two even numbers be $2m$ and $2n$ (using different letters as they are not necessarily consecutive).
2. Product: $(2m)(2n)$
3. Simplify: $4mn$
4. Conclusion: Since 4 is a factor, 4 × (integer product) is always a **multiple of 4**.

You Do 2:

1. Let the consecutive integers be n and $(n + 1)$.
2. Difference of their squares: $(n + 1)^2 - n^2$
3. Expand and simplify: $(n^2 + 2n + 1) - n^2 = 2n + 1$
4. Sum of the two integers: $n + (n + 1) = 2n + 1$
5. Conclusion: Both expressions evaluate to $\mathbf{2n + 1}$, therefore they are always equal.

Challenge 1:

1. Let the four consecutive integers be n, $(n + 1)$, $(n + 2)$, and $(n + 3)$.
2. Set up the sum: $n + (n + 1) + (n + 2) + (n + 3)$
3. Simplify: $4n + 6$
4. Factor out 4 with a remainder: $4n + 4 + 2 = 4(n + 1) + 2$
5. Conclusion: $4(n + 1)$ is a multiple of 4. Adding 2 leaves a remainder of 2, so it is **never a multiple of 4**.

Challenge 2:

1. Expand the first bracket: $(n + 3)^2 = n^2 + 6n + 9$
2. Expand the second bracket: $n(n + 6) = n^2 + 6n$
3. Subtract: $(n^2 + 6n + 9) - (n^2 + 6n)$
4. Simplify: $n^2 + 6n + 9 - n^2 - 6n = 9$
5. Conclusion: $9 = 3^2$. Since the result is always 9, which is a **perfect square**, the statement holds true for all n.