Algebra: Completing the Square & Turning Points

Practice Worksheet (Jan 2022 Paper 1H Style)

I Do

(a) Express $7 + 12x - 3x^2$ in the form $a + b(x + c)^2$ where a, b, and c are integers.

C is the curve with equation $y = 7 + 12x - 3x^2$. The point A is the maximum point on **C**.

(b) Use your answer to part (a) to write down the coordinates of A.

We Do

(a) Express $5 + 12x - 2x^2$ in the form $a + b(x + c)^2$ where a, b, and c are integers.

C is the curve with equation $y = 5 + 12x - 2x^2$. The point P is the maximum point on **C**.

(b) Use your answer to part (a) to write down the coordinates of P.

You Do

(a) Express $11 + 24x - 4x^2$ in the form $a + b(x + c)^2$ where a, b, and c are integers.

C is the curve with equation $y = 11 + 24x - 4x^2$. The point Q is the maximum point on **C**.

(b) Use your answer to part (a) to write down the coordinates of Q.

Challenge

(a) Express $3 + 10x - 5x^2$ in the form $a + b(x + c)^2$ where a, b, and c are integers.

Curve **C** has equation $y = 3 + 10x - 5x^2$. Point M is the maximum point on **C**.

(b) Write down the coordinates of M.

(c) Show algebraically that the line $y = 2x + 1$ intersects the curve **C** at two distinct real points.

Answers

- **I Do:**

(a) $7 - 3(x^2 - 4x) = 7 - 3\left[(x-2)^2 - 4\right] = 7 - 3(x-2)^2 + 12 = \mathbf{19 - 3(x-2)^2}$

$(a = 19, b = -3, c = -2)$

(b) Maximum occurs when $(x-2)^2 = 0 \implies x = 2, y = 19 \implies \mathbf{A(2, 19)}$

- **We Do:**

(a) $5 - 2(x^2 - 6x) = 5 - 2\left[(x-3)^2 - 9\right] = 5 - 2(x-3)^2 + 18 = \mathbf{23 - 2(x-3)^2}$

$(a = 23, b = -2, c = -3)$

(b) Maximum occurs when $(x-3)^2 = 0 \implies x = 3, y = 23 \implies \mathbf{P(3, 23)}$

- **You Do:**

(a) $11 - 4(x^2 - 6x) = 11 - 4\left[(x-3)^2 - 9\right] = 11 - 4(x-3)^2 + 36 = \mathbf{47 - 4(x-3)^2}$

$(a = 47, b = -4, c = -3)$

(b) Maximum occurs when $(x-3)^2 = 0 \implies x = 3, y = 47 \implies \mathbf{Q(3, 47)}$

- **Challenge:**

(a) $3 - 5(x^2 - 2x) = 3 - 5\left[(x-1)^2 - 1\right] = \mathbf{8 - 5(x-1)^2} \quad (a = 8, b = -5, c = -1)$

(b) Coordinates of $M = (\mathbf{1}, \mathbf{8})$

(c) Equate equations: $3 + 10x - 5x^2 = 2x + 1 \implies 5x^2 - 8x - 2 = 0$

Discriminant $b^2 - 4ac = (-8)^2 - 4(5)(-2) = 64 + 40 = 104 > 0$

Since $\Delta > 0$, there are 2 distinct points of intersection.