Algebra: Intersection of Line and Curve

Practice Worksheet (Jun 2024 Paper 2H Style)

I Do

The straight line **L** has equation $x + y = 5$
The curve **C** has equation $2x^2 + 3y^2 = 210$

Find the coordinates of the points where **L** and **C** intersect.
Show clear algebraic working.

We Do

The straight line L has equation

$$x + y = 6$$

The curve C has equation

$$x^2 + 2y^2 = 132$$

Find the coordinates of the points where L and C intersect.

Show clear algebraic working.

You Do

The straight line L has equation

$$x + y = 4$$

The curve C has equation

$$3x^2 + y^2 = 76$$

Find the coordinates of the points where L and C intersect.

Show clear algebraic working.

Challenge

The straight line L has equation

$$2x + y = 5$$

The circle C has equation

$$x^2 + y^2 = 25$$

Find the coordinates of the points where L and C intersect.

Show clear algebraic working.

Answers

- **I Do:**

Rearrange line equation: $y = 5 - x$

Substitute into curve: $2x^2 + 3(5 - x)^2 = 210$

$2x^2 + 3(25 - 10x + x^2) = 210 \implies 5x^2 - 30x - 135 = 0$

Divide by 5: $x^2 - 6x - 27 = 0 \implies (x - 9)(x + 3) = 0$

Critical values: $x = 9$ or $x = -3$

When $x = 9, y = -4$ | When $x = -3, y = 8$

$(\mathbf{9}, \mathbf{-4})$ and $(\mathbf{-3}, \mathbf{8})$

- **We Do:**

Rearrange line equation: $y = 6 - x$

Substitute into curve: $x^2 + 2(6 - x)^2 = 132$

$x^2 + 2(36 - 12x + x^2) = 132 \implies 3x^2 - 24x - 60 = 0$

Divide by 3: $x^2 - 8x - 20 = 0 \implies (x - 10)(x + 2) = 0$

Critical values: $x = 10$ or $x = -2$

When $x = 10, y = -4$ | When $x = -2, y = 8$

$(\mathbf{10}, \mathbf{-4})$ and $(\mathbf{-2}, \mathbf{8})$

- **You Do:**

Rearrange line equation: $y = 4 - x$

Substitute into curve: $3x^2 + (4 - x)^2 = 76$

$3x^2 + (16 - 8x + x^2) = 76 \implies 4x^2 - 8x - 60 = 0$

Divide by 4: $x^2 - 2x - 15 = 0 \implies (x - 5)(x + 3) = 0$

Critical values: $x = 5$ or $x = -3$

When $x = 5, y = -1$ | When $x = -3, y = 7$

$(\mathbf{5}, \mathbf{-1})$ and $(\mathbf{-3}, \mathbf{7})$

- **Challenge:**

Rearrange line equation: $y = 5 - 2x$

Substitute into circle: $x^2 + (5 - 2x)^2 = 25$

$x^2 + (25 - 20x + 4x^2) = 25 \implies 5x^2 - 20x = 0$

Factorise: $5x(x - 4) = 0 \implies x = 0$ or $x = 4$

When $x = 0, y = 5$ | When $x = 4, y = -3$

$(\mathbf{0}, \mathbf{5})$ and $(\mathbf{4}, \mathbf{-3})$