# 3D Geometry: Cuboid Volume & Surface Area

*Practice Worksheet (Edexcel GCSE 1MA1 June 2024 1H Style)*

## I Do

A solid cuboid has a volume of 300 $cm^3$.
The cuboid has a total surface area of 370 $cm^2$.

The length of the cuboid is 20 cm.
The width of the cuboid is greater than the height of the cuboid.

Work out the height of the cuboid.
You must show all your working.

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## We Do

A solid cuboid has a volume of 240 $cm^3$.
The cuboid has a total surface area of 268 $cm^2$.

The length of the cuboid is 10 cm.
The width of the cuboid is greater than the height of the cuboid.

Work out the height of the cuboid.
You must show all your working.

## You Do

A solid cuboid has a volume of 336 $cm^3$.
The cuboid has a total surface area of 320 $cm^2$.

The length of the cuboid is 12 cm.
The width of the cuboid is greater than the height of the cuboid.

Work out the height of the cuboid.
You must show all your working.

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## Challenge

A solid cuboid has a volume of 1152 $cm^3$.
The cuboid has a total surface area of 768 $cm^2$.

The length of the cuboid is 24 cm.
The width of the cuboid is greater than the height of the cuboid.

(a) Work out the height of the cuboid.

(b) Calculate the exact length of the space diagonal (distance between two opposite furthest vertices) of the cuboid.

# Answers & Worked Solutions

- **I Do:**
  1. Volume equation: $V = l \cdot w \cdot h \implies 20wh = 300 \implies wh = 15$.
  2. Surface area equation: $A = 2(lw + lh + wh) = 370 \implies 20w + 20h + 15 = 185$.
  3. Simplify: $20(w + h) = 170 \implies w + h = 8.5 \implies w = 8.5 - h$.
  4. Substitute into $wh = 15$: $(8.5 - h)h = 15 \implies h^2 - 8.5h + 15 = 0$.
  5. Multiply by 2: $2h^2 - 17h + 30 = 0 \implies (2h - 5)(h - 6) = 0$.
  6. $h = 2.5$ cm (with $w = 6$ cm $> h$) or $h = 6$ cm (with $w = 2.5$ cm $< h$).
  7. Since width $>$ height: Height $\mathbf{h = 2.5}$ cm.

- **We Do:**
  1. Volume: $10wh = 240 \implies wh = 24$.
  2. Surface area:
  $2(10w + 10h + 24) = 268 \implies 10(w + h) + 24 = 134 \implies 10(w + h) = 110 \implies w + h = 11$.
  3. Substitute $w = 11 - h$: $(11 - h)h = 24 \implies h^2 - 11h + 24 = 0$.
  4. Factorise: $(h - 3)(h - 8) = 0 \implies h = 3$ cm or $h = 8$ cm.
  5. Since width $>$ height ($w = 8 > h = 3$): Height $\mathbf{h = 3}$ cm.

- **You Do:**
  1. Volume: $12wh = 336 \implies wh = 28$.
  2. Surface area:
  $2(12w + 12h + 28) = 320 \implies 12(w + h) + 28 = 160 \implies 12(w + h) = 132 \implies w + h = 11$.
  3. Substitute $w = 11 - h$: $(11 - h)h = 28 \implies h^2 - 11h + 28 = 0$.
  4. Factorise: $(h - 4)(h - 7) = 0 \implies h = 4$ cm or $h = 7$ cm.
  5. Since width $>$ height ($w = 7 > h = 4$): Height $\mathbf{h = 4}$ cm.

- **Challenge:**
  1. (a) Volume: $24wh = 1152 \implies wh = 48$.
  Surface area:
  $2(24w + 24h + 48) = 768 \implies 24(w + h) + 48 = 384 \implies 24(w + h) = 336 \implies w + h = 14$.
  Substitute $w = 14 - h$: $h^2 - 14h + 48 = 0 \implies (h - 6)(h - 8) = 0$.
  Since $w > h \implies w = 8$ cm and Height $\mathbf{h = 6}$ cm.
  2. (b) Space diagonal
  $D = \sqrt{l^2 + w^2 + h^2} = \sqrt{24^2 + 8^2 + 6^2} = \sqrt{576 + 64 + 36} = \sqrt{676} = \mathbf{26}$ cm.