Probability Number Properties Worksheet

Sums and Products of Odds & Evens (Without Replacement)

Reference: Edexcel GCSE 1MA1 / Higher Tier / Paper 1H / Q14

I Do

There are nine balls labelled 1 to 9 in a box.

Lee will take at random two balls from the box.

Lee says,

> "The probability that the sum of the numbers on the two balls will be an even number is greater than the probability that the product of the numbers will be an even number."

Is Lee correct? You must show how you get your answer.

(Total for Question 1 is 4 marks)

We Do

There are ten counters labelled 1 to 10 in a bag.

Sam takes at random two counters from the bag without replacement.

Sam says,

> "The probability that the product of the two numbers is even is more than twice the probability that the sum of the two numbers is even."

Is Sam correct? You must show how you get your answer.

(Total for Question 2 is 4 marks)

You Do

There are seven cards numbered 1 to 7 in a box.

Mia takes two cards at random without replacement.

Mia claims,

> "The probability that the product of the numbers on the two cards is odd is less than the probability that the sum of the numbers is odd."

Is Mia correct? You must show how you get your answer.

(Total for Question 3 is 4 marks)

Challenge

A box contains n cards labelled 1 to n, where n is an odd integer ($n \geq 3$).

Two cards are taken at random without replacement.

Show that the probability that the product of the numbers on the two cards is even is given by

$$\frac{3n-1}{4n}$$

(Total for Question 4 is 5 marks)

Answers & Worked Solutions

• **I Do:**

1. Count Odds and Evens from 1 to 9:
Odds: $\{1,3,5,7,9\}$ ***(5 odd balls), Evens:*** $\{2,4,6,8\}$ ***(4 even balls). Total = 9.***

2. Find P(Sum is Even):
Sum is even $\implies$ ***(Odd, Odd) or (Even, Even)***
$P(\textbf{Odd, Odd}) = \frac{5}{9} \times \frac{4}{8} = \frac{20}{72}$
$P(\textbf{Even, Even}) = \frac{4}{9} \times \frac{3}{8} = \frac{12}{72}$
$P(\textbf{Sum is Even}) = \frac{20}{72} + \frac{12}{72} = \frac{32}{72}$

3. Find P(Product is Even):
Product is odd only if $(\textbf{Odd, Odd}) \implies P(\textbf{Product is Odd}) = \frac{20}{72}$
$P(\textbf{Product is Even}) = 1 - \frac{20}{72} = \frac{52}{72}$

4. Compare probabilities:
$\frac{32}{72} < \frac{52}{72}$***, so*** $P(\textbf{Sum is Even}) < P(\textbf{Product is Even})$.
Conclusion: Lee is **incorrect**.

• **We Do:**

1. Count Odds and Evens from 1 to 10:
5 Odds, 5 Evens. Total = 10.

2. Find P(Sum is Even):
$P(\textbf{Odd, Odd}) = \frac{5}{10} \times \frac{4}{9} = \frac{20}{90}$
$P(\textbf{Even, Even}) = \frac{5}{10} \times \frac{4}{9} = \frac{20}{90} \implies P(\textbf{Sum is Even}) = \frac{40}{90} = \frac{4}{9}$

3. Find P(Product is Even):
$P(\textbf{Product is Even}) = 1 - P(\textbf{Odd, Odd}) = 1 - \frac{20}{90} = \frac{70}{90} = \frac{7}{9}$

4. Compare $P(\textbf{Product is Even})$ ***with*** $2 \times P(\textbf{Sum is Even})$***:***
$2 \times P(\textbf{Sum is Even}) = 2 \times \frac{4}{9} = \frac{8}{9}$
Since $\frac{7}{9} < \frac{8}{9}$***, the product probability is NOT more than twice the sum probability.***

Conclusion: Sam is **incorrect**.

• **You Do:**

1. Count Odds and Evens from 1 to 7:
4 Odds, 3 Evens. Total = 7.

2. Find P(Product is Odd):
$P(\textbf{Product is Odd}) = P(\textbf{Odd, Odd}) = \frac{4}{7} \times \frac{3}{6} = \frac{12}{42}$

3. Find P(Sum is Odd):
Sum is odd $\implies$ $(\textbf{Odd, Even})$ ***or*** $(\textbf{Even, Odd})$
$P(\textbf{Odd, Even}) = \frac{4}{7} \times \frac{3}{6} = \frac{12}{42}$
$P(\textbf{Even, Odd}) = \frac{3}{7} \times \frac{4}{6} = \frac{12}{42} \implies P(\textbf{Sum is Odd}) = \frac{24}{42}$

4. Compare:
Since $\frac{12}{42} < \frac{24}{42}$, $P(\textbf{Product is Odd}) < P(\textbf{Sum is Odd})$.

Conclusion: Mia is **correct**.

• **Challenge:**

1. For n ***cards where*** n ***is odd:***
Number of odd cards = $\frac{n+1}{2}$, ***Number of even cards =*** $\frac{n-1}{2}$.

2. $P(\textbf{Product is Odd}) = P(\textbf{both cards are odd})$***:***
$P(\textbf{both odd}) = \frac{\frac{n+1}{2}}{n} \times \frac{\frac{n+1}{2}-1}{n-1} = \frac{\frac{n+1}{2}}{n} \times \frac{\frac{n-1}{2}}{n-1} = \frac{(n+1)(n-1)}{4n(n-1)} = \frac{n+1}{4n}$

3. $P(\textbf{Product is Even}) = 1 - P(\textbf{Product is Odd})$***:***
$P(\textbf{Product is Even}) = 1 - \frac{n+1}{4n} = \frac{4n-(n+1)}{4n} = \frac{3n-1}{4n}$ ■