Composite Functions & Equations Worksheet

Composite Functions, Algebraic Proofs & Solvability

Reference: Edexcel GCSE 1MA1 / Higher Tier / Paper 1 / Q17

I Do

$g(x) = 1 - 3x$ and $h(x) = 2x^2 - 1$

Show that $3gh(x) + hg(x) = 0$ has just one solution for x.

(Total for Question 1 is 5 marks)

We Do

$f(x) = 2 - 4x$ and $g(x) = 3x^2 - 2$

Show that $4fg(x) + gf(x) = 0$ has just one solution for x.

(Total for Question 2 is 5 marks)

You Do

$f(x) = 5 - 2x$ and $g(x) = 3x^2 + 1$

Show that $2fg(x) + gf(x) = 0$ has just one solution for x.

(Total for Question 3 is 5 marks)

Challenge

$g(x) = 2x - 1$ and $h(x) = x^2 + 4x + k$

Given that $hg(x) = 0$ has just one solution for x, find the value of k.

(Total for Question 4 is 5 marks)

Answers & Worked Solutions

• **I Do:**

1. Find $gh(x)$***:*** $g(2x^2-1) = 1-3(2x^2-1) = 4-6x^2$

2. Find $3gh(x)$***:*** $3(4-6x^2) = 12-18x^2$

3. Find $hg(x)$***:*** $h(1-3x) = 2(1-3x)^2-1 = 2(1-6x+9x^2)-1 = 18x^2-12x+1$

4. Set up equation $3gh(x)+hg(x)=0$***:***

$(12-18x^2)+(18x^2-12x+1)=0 \implies 13-12x=0$

5. Solve: $12x=13 \implies x=\frac{13}{12}$

Since the x^2 ***terms cancel out, the equation reduces to a linear equation (***$13-12x=0$***), which has exactly one solution (***$x=\frac{13}{12}$***).***

• **We Do:**

1. Find $fg(x)$***:*** $f(3x^2-2) = 2-4(3x^2-2) = 10-12x^2$

2. Find $4fg(x)$***:*** $4(10-12x^2) = 40-48x^2$

3. Find $gf(x)$***:*** $g(2-4x) = 3(2-4x)^2-2 = 3(4-16x+16x^2)-2 = 48x^2-48x+10$

4. Set up equation $4fg(x)+gf(x)=0$***:***

$(40-48x^2)+(48x^2-48x+10)=0 \implies 50-48x=0$

5. Solve: $48x=50 \implies x=\frac{25}{24}$

Since x^2 ***terms cancel out, the equation is linear (***$50-48x=0$***), which has just one solution (***$x=\frac{25}{24}$***).***

• **You Do:**

1. Find $fg(x)$***:*** $f(3x^2+1) = 5-2(3x^2+1) = 3-6x^2$

2. Find $2fg(x)$***:*** $2(3-6x^2) = 6-12x^2$

3. Find $gf(x)$***:*** $g(5-2x) = 3(5-2x)^2+1 = 3(25-20x+4x^2)+1 = 12x^2-60x+76$

4. Set up equation $2fg(x)+gf(x)=0$***:***

$(6-12x^2)+(12x^2-60x+76)=0 \implies 82-60x=0$

5. Solve: $60x=82 \implies x=\frac{41}{30}$

Since x^2 ***terms cancel out, the equation is linear (***$82-60x=0$***), having just one solution (***$x=\frac{41}{30}$***).***

• **Challenge:**

1. Find $hg(x)$***:*** $h(2x-1) = (2x-1)^2+4(2x-1)+k$

2. Expand: $hg(x) = (4x^2-4x+1)+(8x-4)+k = 4x^2+4x+(k-3)$

3. For $4x^2+4x+(k-3)=0$ ***to have just one solution, discriminant*** $b^2-4ac=0$***:***

$4^2-4(4)(k-3)=0 \implies 16-16(k-3)=0$

4. Solve for k***:*** $16(k-3)=16 \implies k-3=1 \implies k=4$

Answer: $k=4$