Functions: Inverse & Composite Functions

Practice Worksheet (Edexcel GCSE 1MA1 Nov 2024 1H Style)

I Do

$$f(x) = \frac{5x - 3}{4}$$

(a) Find $f^{-1}(x)$.

For all values of x,

$$g(x) = (x - 1)^2 \qquad \text{and} \qquad h(x) = 1 - 2x$$

(b) Work out the value of $gh(5)$.

We Do

$$f(x) = \frac{3x + 5}{2}$$

(a) Find $f^{-1}(x)$.

For all values of x,

$$g(x) = (x + 2)^2 \qquad \text{and} \qquad h(x) = 3 - 4x$$

(b) Work out the value of $gh(2)$.

You Do

$$f(x) = \frac{4x - 7}{3}$$

(a) Find $f^{-1}(x)$.

For all values of x,

$$g(x) = (x - 3)^2 \qquad \text{and} \qquad h(x) = 2 - 3x$$

(b) Work out the value of $gh(4)$.

Challenge

For all values of x,

$$f(x) = 2x + 1 \qquad g(x) = x^2 - 3 \qquad h(x) = 3x - 2$$

(a) Find an expression for $fg(x)$ in terms of x. Give your answer in its simplest form.

(b) Solve the equation $gf(x) = h(x)$.

Answers & Worked Solutions

- **I Do:**
  (a) Set $y = \frac{5x-3}{4} \implies 4y = 5x - 3 \implies 5x = 4y + 3 \implies x = \frac{4y+3}{5}$.
  So, $\mathbf{f^{-1}(x) = \frac{4x+3}{5}}$.
  (b) $h(5) = 1 - 2(5) = 1 - 10 = -9$.
  $gh(5) = g(-9) = (-9 - 1)^2 = (-10)^2 = \mathbf{100}$.

- **We Do:**
  (a) Set $y = \frac{3x+5}{2} \implies 2y = 3x + 5 \implies 3x = 2y - 5 \implies x = \frac{2y-5}{3}$.
  So, $\mathbf{f^{-1}(x) = \frac{2x-5}{3}}$.
  (b) $h(2) = 3 - 4(2) = 3 - 8 = -5$.
  $gh(2) = g(-5) = (-5 + 2)^2 = (-3)^2 = \mathbf{9}$.

- **You Do:**
  (a) Set $y = \frac{4x-7}{3} \implies 3y = 4x - 7 \implies 4x = 3y + 7 \implies x = \frac{3y+7}{4}$.
  So, $\mathbf{f^{-1}(x) = \frac{3x+7}{4}}$.
  (b) $h(4) = 2 - 3(4) = 2 - 12 = -10$.
  $gh(4) = g(-10) = (-10 - 3)^2 = (-13)^2 = \mathbf{169}$.

- **Challenge:**
  (a) $fg(x) = f(g(x)) = f(x^2 - 3) = 2(x^2 - 3) + 1 = 2x^2 - 6 + 1 = \mathbf{2x^2 - 5}$.
  (b) $gf(x) = g(f(x)) = g(2x + 1) = (2x + 1)^2 - 3 = (4x^2 + 4x + 1) - 3 = 4x^2 + 4x - 2$.
  Set $gf(x) = h(x) \implies 4x^2 + 4x - 2 = 3x - 2 \implies 4x^2 + x = 0$.
  Factorise: $x(4x + 1) = 0 \implies \mathbf{x = 0}$ or $\mathbf{x = -\frac{1}{4}}$.