Probability: Multi-Stage Combined Events

Practice Worksheet (Edexcel GCSE 1MA1 Nov 2024 1H Style)

I Do

In the semi-finals of a chess tournament,
player **A** will play player **B**
and player **C** will play player **D**.

The two winners will then play each other in the final.

The probability that player **A** will win against player **B** is 0.6
The probability that player **A** will win against player **C** is 0.5
The probability that player **A** will win against player **D** is 0.3
The probability that player **C** will win against player **D** is 0.2

Work out the probability that player **A** will win the chess tournament.

We Do

In the semi-finals of a chess tournament,
player **A** will play player **B**
and player **C** will play player **D**.

The two winners will then play each other in the final.

The probability that player **A** will win against player **B** is 0.7
The probability that player **A** will win against player **C** is 0.6
The probability that player **A** will win against player **D** is 0.4
The probability that player **C** will win against player **D** is 0.3

Work out the probability that player **A** will win the chess tournament.

You Do

In the semi-finals of a chess tournament,
player **A** will play player **B**
and player **C** will play player **D**.

The two winners will then play each other in the final.

The probability that player **A** will win against player **B** is 0.8
The probability that player **A** will win against player **C** is 0.4
The probability that player **A** will win against player **D** is 0.5
The probability that player **C** will win against player **D** is 0.6

Work out the probability that player **A** will win the chess tournament.

Challenge

In the semi-finals of a tennis tournament,
player **A** will play player **B**
and player **C** will play player **D**.

The two winners will then play each other in the final.

The probability that player **A** will win against player **B** is 0.75
The probability that player **A** will win against player **C** is 0.6
The probability that player **A** will win against player **D** is 0.4
The probability that player **C** will win against player **D** is 0.4

(a) Work out the probability that player **A** will win the tournament.

(b) Given that player **A** wins the tournament, find the conditional probability that player **A** played against player **C** in the final.

Answers & Worked Solutions

- **I Do:**
  1. For player **A** to win the tournament, **A** must first beat **B** in Semi-final 1: $P(\text{A beats B}) = 0.6$.
  2. Case 1 (**C** wins Semi-final 2):
     - $P(\text{C beats D}) = 0.2$
     - Final is **A** vs **C**: $P(\text{A beats C}) = 0.5$
     - $P(\text{Path 1}) = 0.6 \times 0.2 \times 0.5 = 0.06$
  3. Case 2 (**D** wins Semi-final 2):
     - $P(\text{D beats C}) = 1 - 0.2 = 0.8$
     - Final is **A** vs **D**: $P(\text{A beats D}) = 0.3$
     - $P(\text{Path 2}) = 0.6 \times 0.8 \times 0.3 = 0.144$
  4. Total Probability $= 0.06 + 0.144 = \mathbf{0.204}$.

- **We Do:**
  1. $P(\text{A beats B}) = 0.7$.
  2. Case 1 (**C** wins Semi 2): $0.7 \times 0.3 \times 0.6 = 0.126$.
  3. Case 2 (**D** wins Semi 2): $0.7 \times (1 - 0.3) \times 0.4 = 0.7 \times 0.7 \times 0.4 = 0.196$.
  4. Total Probability $= 0.126 + 0.196 = \mathbf{0.322}$.

- **You Do:**
  1. $P(\text{A beats B}) = 0.8$.
  2. Case 1 (**C** wins Semi 2): $0.8 \times 0.6 \times 0.4 = 0.192$.
  3. Case 2 (**D** wins Semi 2): $0.8 \times (1 - 0.6) \times 0.5 = 0.8 \times 0.4 \times 0.5 = 0.160$.
  4. Total Probability $= 0.192 + 0.160 = \mathbf{0.352}$.

- **Challenge:**
  1. (a) $P(\text{Path A-C-A}) = 0.75 \times 0.4 \times 0.6 = 0.18$.
  $P(\text{Path A-D-A}) = 0.75 \times (1 - 0.4) \times 0.4 = 0.75 \times 0.6 \times 0.4 = 0.18$.
  $P(\text{A wins tournament}) = 0.18 + 0.18 = \mathbf{0.36}$.
  2. (b) Conditional Probability:

$$P(\text{Played C} \mid \text{A won}) = \frac{P(\text{A beats C in final})}{P(\text{A wins tournament})} = \frac{0.18}{0.36} = \mathbf{0.5} \text{ (or } \frac{\mathbf{1}}{\mathbf{2}}\text{)}.$$