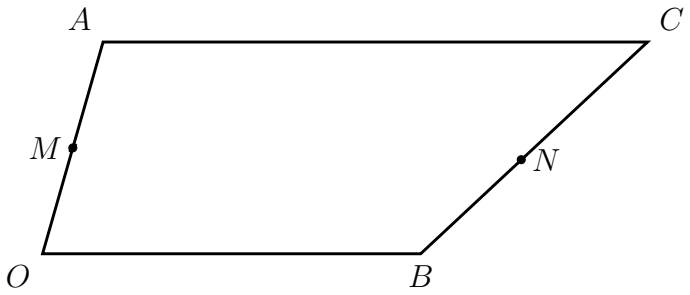
Vectors: Vector Geometry & Ratios

Practice Worksheet (Edexcel GCSE 1MA1 June 2024 2H Style)

I Do

The diagram shows quadrilateral $OACB$.



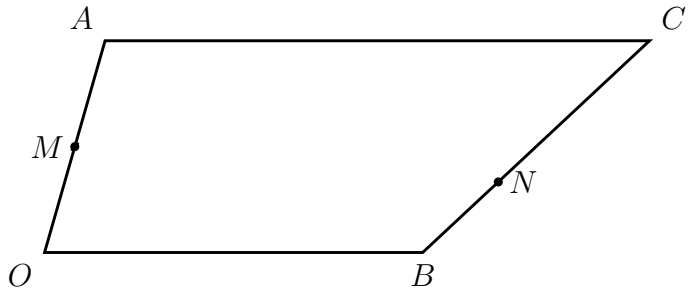
M is the midpoint of OA.
N is the point on BC such that $BN : NC = 4 : 5$

$$\vec{OA} = \mathbf{a} \qquad \vec{OB} = \mathbf{b} \qquad \vec{AC} = k\mathbf{b} \quad \text{where } k \text{ is a positive integer.}$$

Express $\vec{MN}$ in terms of k, $\mathbf{a}$ and $\mathbf{b}$.
Give your answer in its simplest form.

We Do

The diagram shows quadrilateral $OACB$.



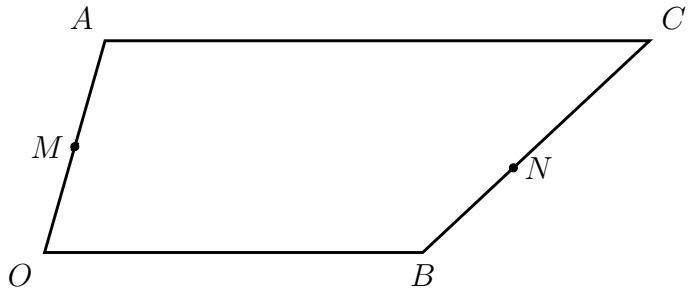
M is the midpoint of OA.
N is the point on BC such that $BN : NC = 1 : 2$

$$\vec{OA} = \mathbf{a} \qquad \vec{OB} = \mathbf{b} \qquad \vec{AC} = k\mathbf{b} \quad \text{where } k \text{ is a positive integer.}$$

Express $\vec{MN}$ in terms of k, $\mathbf{a}$ and $\mathbf{b}$.
Give your answer in its simplest form.

You Do

The diagram shows quadrilateral $OACB$.



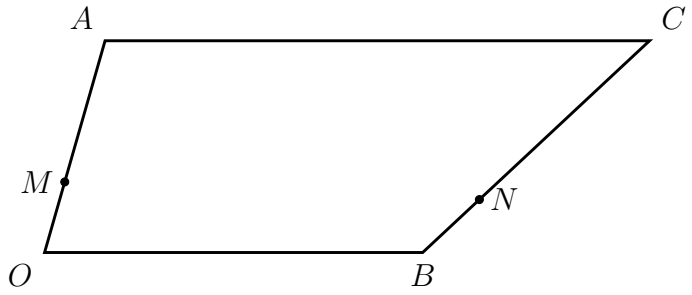
M is the midpoint of OA.
N is the point on BC such that $BN : NC = 2 : 3$

$$\vec{OA} = \mathbf{a} \qquad \vec{OB} = \mathbf{b} \qquad \vec{AC} = k\mathbf{b} \quad \text{where } k \text{ is a positive integer.}$$

Express $\vec{MN}$ in terms of k, $\mathbf{a}$ and $\mathbf{b}$.
Give your answer in its simplest form.

Challenge

The diagram shows quadrilateral $OACB$.



M is a point on OA such that $OM : MA = 1 : 2$.
N is a point on BC such that $BN : NC = 1 : 3$.

$$\vec{OA} = \mathbf{a} \qquad \vec{OB} = \mathbf{b} \qquad \vec{AC} = k\mathbf{b} \quad \text{where } k \text{ is a positive integer.}$$

(a) Express $\vec{MN}$ in terms of k, $\mathbf{a}$ and $\mathbf{b}$.

(b) Given that $\vec{MN}$ is parallel to the vector $-\mathbf{a} + 12\mathbf{b}$, find the value of k.

Answers & Worked Solutions

- **I Do:**
  1. Express $\vec{BC}$:
  $$\vec{BC} = \vec{BO} + \vec{OA} + \vec{AC} = -\mathbf{b} + \mathbf{a} + k\mathbf{b} = \mathbf{a} + (k-1)\mathbf{b}$$
  2. Express $\vec{BN}$:
  $$\vec{BN} = \frac{4}{9}\vec{BC} = \frac{4}{9}(\mathbf{a} + (k-1)\mathbf{b})$$
  3. Express $\vec{MN}$ via path $M \to O \to B \to N$:
  $$\vec{MN} = \vec{MO} + \vec{OB} + \vec{BN} = -\frac{1}{2}\mathbf{a} + \mathbf{b} + \frac{4}{9}\mathbf{a} + \frac{4}{9}(k-1)\mathbf{b}$$
  4. Combine coefficients:
  $$\text{Coefficient of } \mathbf{a} : -\frac{1}{2} + \frac{4}{9} = -\frac{1}{18}$$
  $$\text{Coefficient of } \mathbf{b} : 1 + \frac{4}{9}k - \frac{4}{9} = \frac{5}{9} + \frac{4}{9}k = \frac{4k+5}{9}$$
  5. Final Answer: $\vec{MN} = -\mathbf{\frac{1}{18}}\mathbf{a} + \mathbf{\frac{4k+5}{9}}\mathbf{b}$.

- **We Do:**
  1. $\vec{BC} = \mathbf{a} + (k-1)\mathbf{b}$.
  2. $\vec{BN} = \frac{1}{3}\vec{BC} = \frac{1}{3}\mathbf{a} + \frac{1}{3}(k-1)\mathbf{b}$.
  3. $\vec{MN} = -\frac{1}{2}\mathbf{a} + \mathbf{b} + \frac{1}{3}\mathbf{a} + \frac{1}{3}(k-1)\mathbf{b}$.
  4. $\mathbf{a}$ term: $-\frac{1}{2} + \frac{1}{3} = -\frac{1}{6}$.
  5. $\mathbf{b}$ term: $1 + \frac{1}{3}k - \frac{1}{3} = \frac{2}{3} + \frac{1}{3}k = \frac{k+2}{3}$.
  6. Final Answer: $\vec{MN} = -\mathbf{\frac{1}{6}}\mathbf{a} + \mathbf{\frac{k+2}{3}}\mathbf{b}$.

- **You Do:**
  1. $\vec{BC} = \mathbf{a} + (k-1)\mathbf{b}$.
  2. $\vec{BN} = \frac{2}{5}\vec{BC} = \frac{2}{5}\mathbf{a} + \frac{2}{5}(k-1)\mathbf{b}$.
  3. $\vec{MN} = -\frac{1}{2}\mathbf{a} + \mathbf{b} + \frac{2}{5}\mathbf{a} + \frac{2}{5}(k-1)\mathbf{b}$.
  4. $\mathbf{a}$ term: $-\frac{1}{2} + \frac{2}{5} = -\frac{1}{10}$.
  5. $\mathbf{b}$ term: $1 + \frac{2}{5}k - \frac{2}{5} = \frac{3}{5} + \frac{2}{5}k = \frac{2k+3}{5}$.
  6. Final Answer: $\vec{MN} = -\mathbf{\frac{1}{10}}\mathbf{a} + \mathbf{\frac{2k+3}{5}}\mathbf{b}$.

- **Challenge:**
  1. (a) $\vec{MO} = -\frac{1}{3}\mathbf{a}$.
  $\vec{BN} = \frac{1}{4}\vec{BC} = \frac{1}{4}\mathbf{a} + \frac{1}{4}(k-1)\mathbf{b}$.
  $\vec{MN} = -\frac{1}{3}\mathbf{a} + \mathbf{b} + \frac{1}{4}\mathbf{a} + \frac{1}{4}(k-1)\mathbf{b} = -\mathbf{\frac{1}{12}}\mathbf{a} + \mathbf{\frac{k+3}{4}}\mathbf{b}$.
  2. (b) $\vec{MN} \parallel (-\mathbf{a} + 12\mathbf{b}) \implies -\frac{1}{12}\mathbf{a} + \frac{k+3}{4}\mathbf{b} = c(-\mathbf{a} + 12\mathbf{b})$.
  Comparing $\mathbf{a}$ coefficients: $c = \frac{1}{12}$.
  Comparing $\mathbf{b}$ coefficients: $\frac{k+3}{4} = 12c = 12\left(\frac{1}{12}\right) = 1$.
  $k + 3 = 4 \implies \mathbf{k = 1}$.