Turning Points of Quadratic Curves Practice

Based on Edexcel 1MA1 / June 2024 / Paper 2H / Q20

Question 1

The curve C has equation $y = 2x^2 - 12x + 7$.

Find the coordinates of the turning point on C.

You must show all your working.

(Total for Question 1 is 3 marks)

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Question 2

The curve C has equation $y = 3x^2 - 18x + 11$.

Find the coordinates of the turning point on C.

You must show all your working.

(Total for Question 2 is 3 marks)

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Question 3

The curve C has equation $y = 2x^2 + 16x - 5$.

Find the coordinates of the turning point on C.

You must show all your working.

(Total for Question 3 is 3 marks)

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Question 4

The curve C has equation $y = 4x^2 - 20x + 3$.

Find the coordinates of the turning point on C.

You must show all your working.

(Total for Question 4 is 3 marks)

Model Answers and Human-Written Solutions

Turning Points and Completing the Square

Question 1: $y = 2x^2 - 12x + 7$

Method 1: Completing the Square

1. Factor out 2 from the x-terms:

$$y = 2(x^2 - 6x) + 7$$

2. Complete the square for the expression inside the bracket:

$$y = 2\left[(x-3)^2 - 3^2\right] + 7 = 2\left[(x-3)^2 - 9\right] + 7$$

3. Expand by multiplying 2 back through:

$$y = 2(x-3)^2 - 18 + 7 = 2(x-3)^2 - 11$$

4. The minimum turning point occurs when $(x-3) = 0 \implies x = 3$, giving $y = -11$.

$$\text{Turning Point} = (3, -11)$$

Method 2: Calculus

1. Differentiate with respect to x: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 4x - 12$.

2. Set $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 \implies 4x - 12 = 0 \implies x = 3$.

3. Substitute $x = 3$ into the curve equation:

$$y = 2(3)^2 - 12(3) + 7 = 18 - 36 + 7 = -11 \implies (3, -11)$$

Question 2: $y = 3x^2 - 18x + 11$

1. $y = 3(x^2 - 6x) + 11 = 3\left[(x-3)^2 - 9\right] + 11$

2. $y = 3(x-3)^2 - 27 + 11 = 3(x-3)^2 - 16$

3. Turning Point $= (3, -16)$

Question 3: $y = 2x^2 + 16x - 5$

1. $y = 2(x^2 + 8x) - 5 = 2\left[(x+4)^2 - 16\right] - 5$
2. $y = 2(x+4)^2 - 32 - 5 = 2(x+4)^2 - 37$
3. Turning Point $= (-4, -37)$

Question 4: $y = 4x^2 - 20x + 3$

1. $y = 4\left(x^2 - 5x\right) + 3 = 4\left[\left(x - \frac{5}{2}\right)^2 - \frac{25}{4}\right] + 3$
2. $y = 4\left(x - \frac{5}{2}\right)^2 - 25 + 3 = 4\left(x - \frac{5}{2}\right)^2 - 22$
3. Turning Point $= \left(\frac{5}{2}, -22\right)$ or $(2.5, -22)$