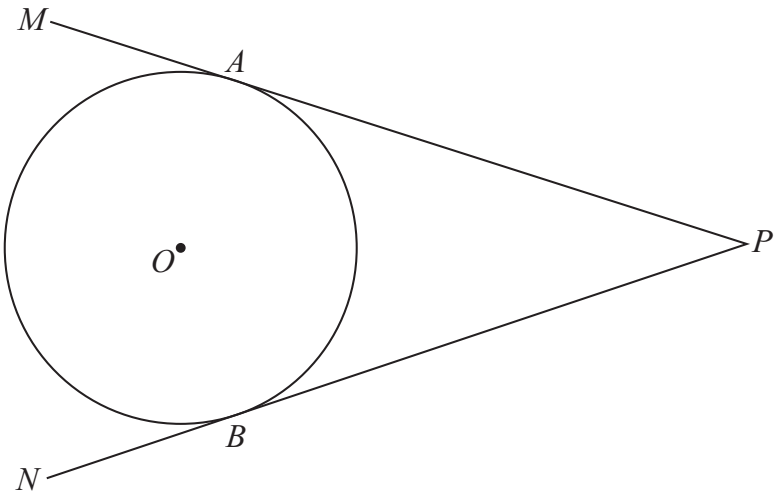
Equal Tangents and Congruence

I Do

A and B are points on a circle, centre O.



MAP and NBP are tangents to the circle.

Prove that $AP = BP$

(Total for Question 22 is 4 marks)

Equal Tangents and Congruence

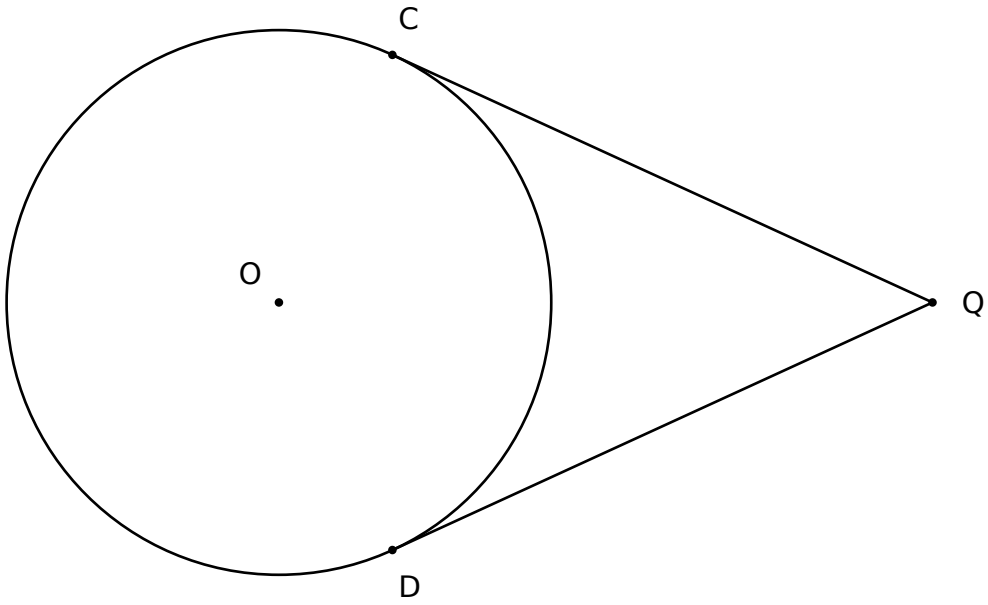
We Do

C and D are points on a circle with centre O.

QC and QD are tangents to the circle.

Prove that QC = QD. Use triangle congruence in your proof.

Not accurately drawn



Show all your working and give reasons.

(Total: 4 marks)

Equal Tangents and Congruence

You Do 1

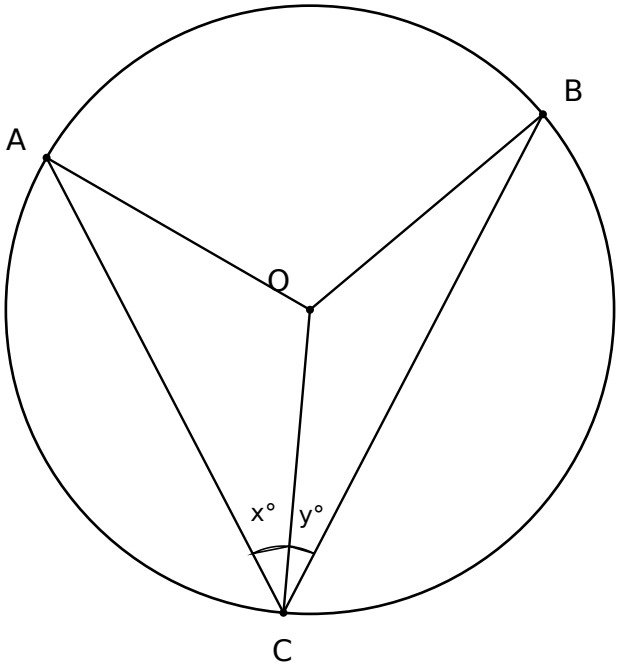
A, B and C lie on a circle with centre O. O lies inside angle ACB.

Let angle ACO = x° and angle OCB = y°.

Prove that the smaller angle AOB is twice angle ACB.

Derive the result using isosceles triangles; do not simply quote the theorem.

Not accurately drawn



Show all your working and give reasons.

(Total: 4 marks)

Equal Tangents and Congruence

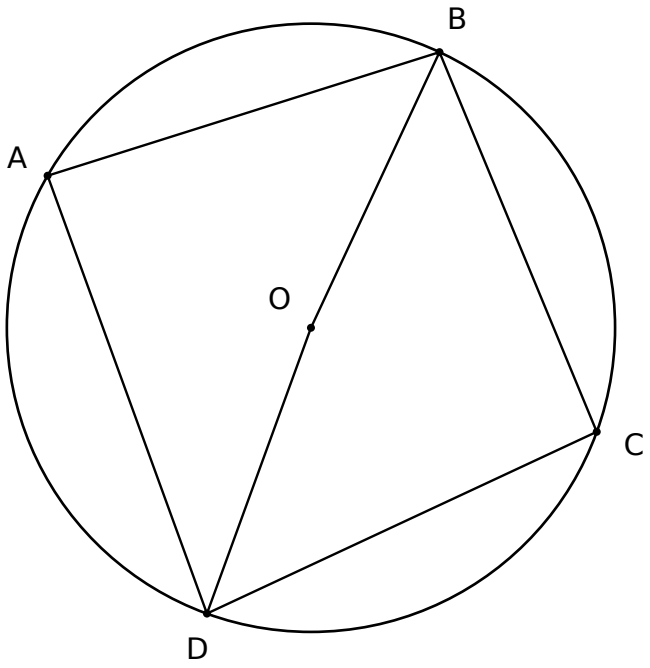
You Do 2

A, B, C and D lie in this order on a circle with centre O.

Prove that angle BAD + angle BCD = 180°.

Use the angle-at-the-centre theorem, rather than quoting the cyclic-quadrilateral theorem.

Not accurately drawn



Show all your working and give reasons.

(Total: 4 marks)

Equal Tangents and Congruence

Challenge 1

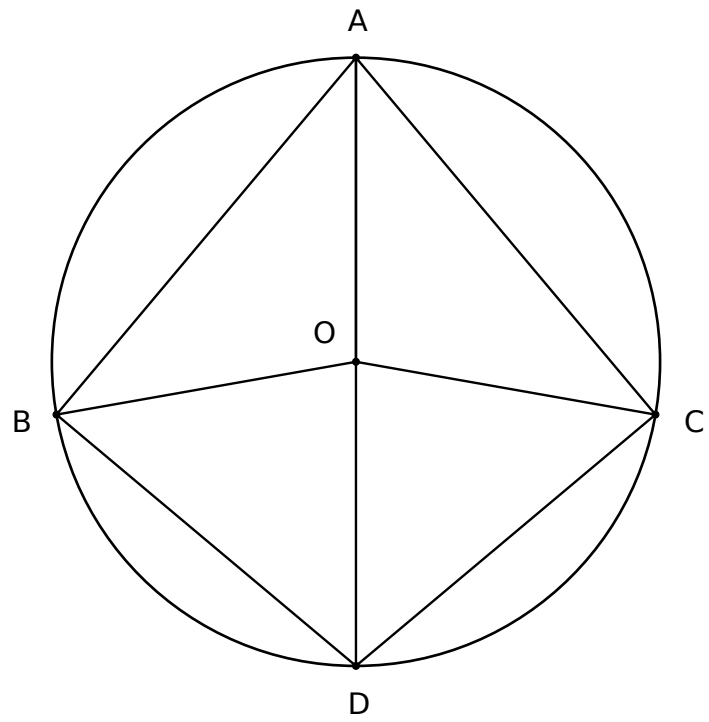
A, B, C and D lie on a circle with centre O. AB = AC.

D lies on the arc BC that does not contain A.

Prove that angle ADB = angle ADC.

Use triangle congruence and the angle-at-the-centre theorem.

Not accurately drawn



Show all your working and give reasons.

(Total: 6 marks)

Equal Tangents and Congruence

Challenge 2

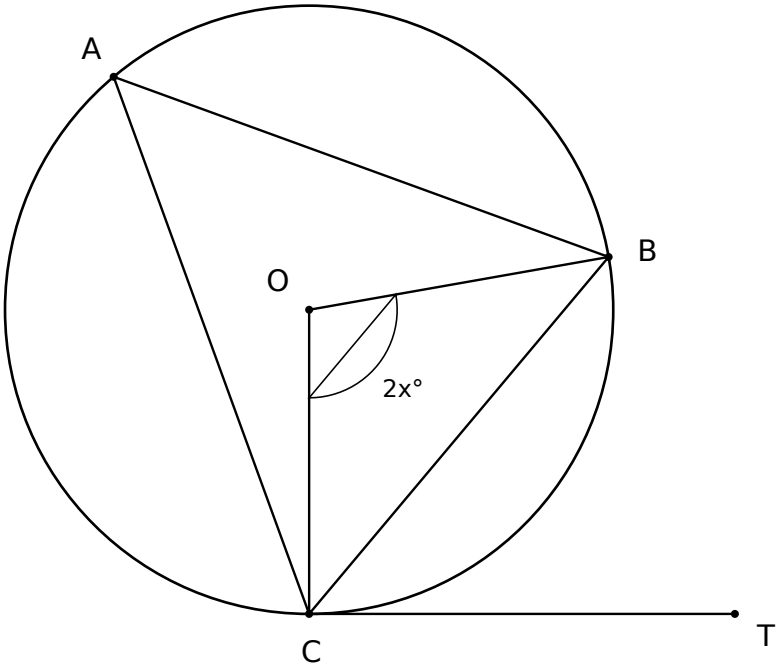
A, B and C lie on a circle with centre O. CT is a tangent at C.

The smaller angle BOC = 2x°.

Prove that angle BCT = angle BAC.

Use radii and the angle-at-the-centre theorem, not the alternate segment theorem.

Not accurately drawn



Show all your working and give reasons.

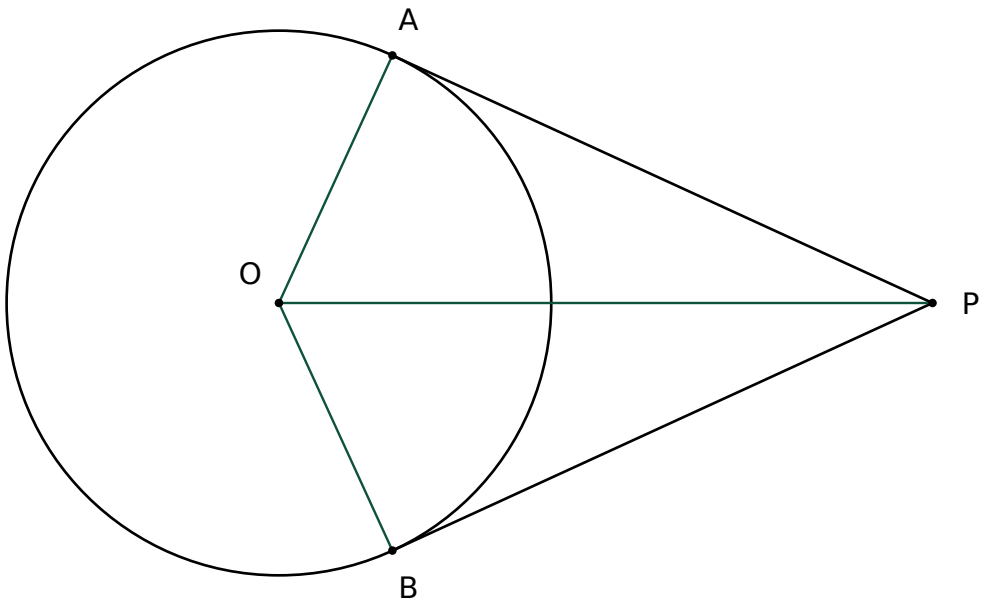
(Total: 6 marks)

Equal Tangents and Congruence

I Do — Worked Solution

1. Join OA, OB and OP.
2. Angle OAP = angle OBP = 90° (radius perpendicular to tangent).
3. OA = OB (radii), and OP is the common hypotenuse.
4. Triangles OAP and OBP are congruent by RHS.
5. Corresponding sides are equal, so PA = PB.

Not accurately drawn



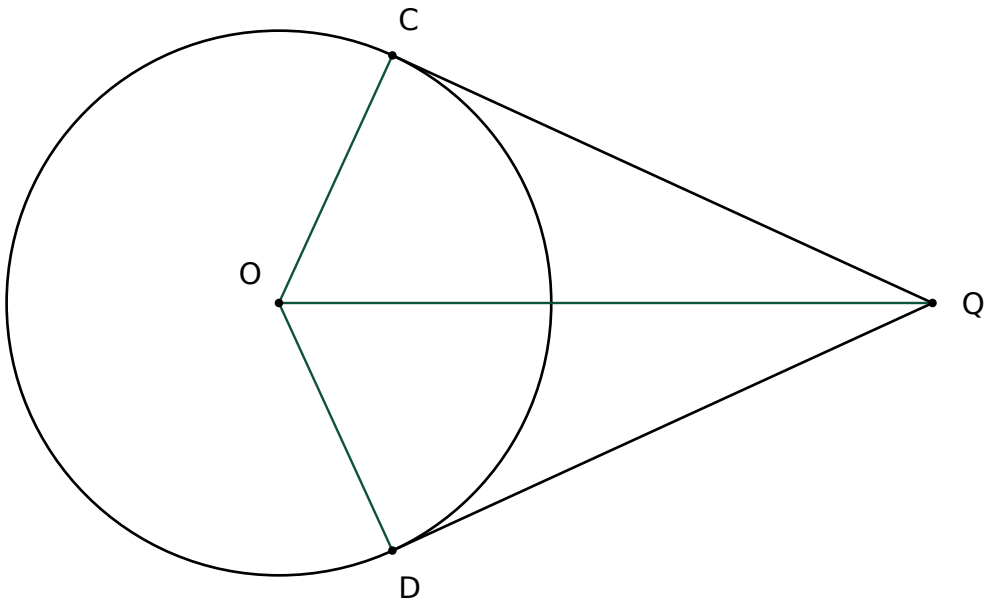
Green dashed lines show added constructions.

Equal Tangents and Congruence

We Do — Worked Solution

1. Join OC, OD and OQ.
2. Angle OCQ = angle ODQ = 90° (radius perpendicular to tangent).
3. OC = OD (radii), and OQ is the common hypotenuse.
4. Triangles OCQ and ODQ are congruent by RHS.
5. Corresponding sides are equal, so QC = QD.

Not accurately drawn



Green dashed lines show added constructions.

Equal Tangents and Congruence

You Do 1 — Worked Solution

1. OA = OC, so angle OAC = angle ACO = x°.

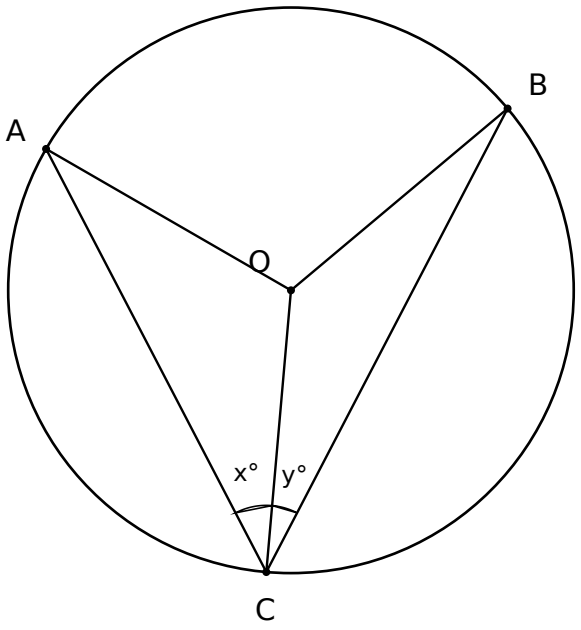
2. OB = OC, so angle OBC = angle OCB = y°.

3. Angle AOC = (180 − 2x)° and angle COB = (180 − 2y)°.

4. Angles around O sum to 360°, so the smaller angle AOB = (2x + 2y)°.

5. Angle ACB = (x + y)°. Hence angle AOB = 2 × angle ACB.

Not accurately drawn



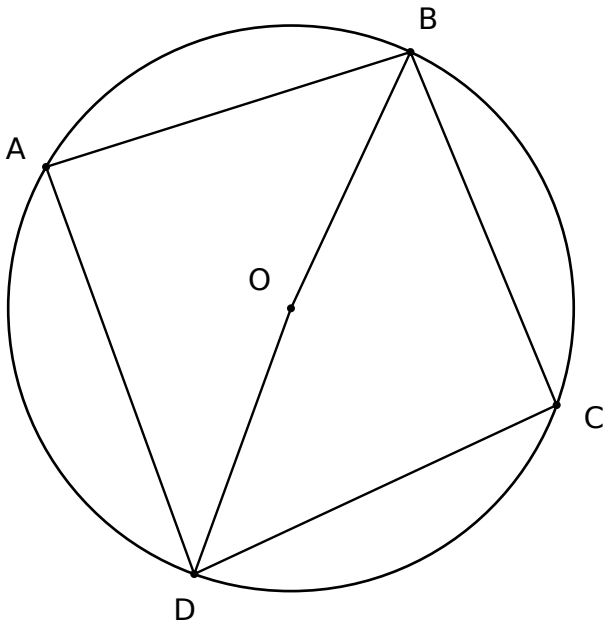
Green dashed lines show added constructions.

Equal Tangents and Congruence

You Do 2 — Worked Solution

1. Angle BAD is half the central angle subtended by the arc BCD.
2. Angle BCD is half the central angle subtended by the other arc BAD.
3. These two central angles, one possibly reflex, add to 360°.
4. Therefore angle BAD + angle BCD = 360°/2 = 180°.

Not accurately drawn



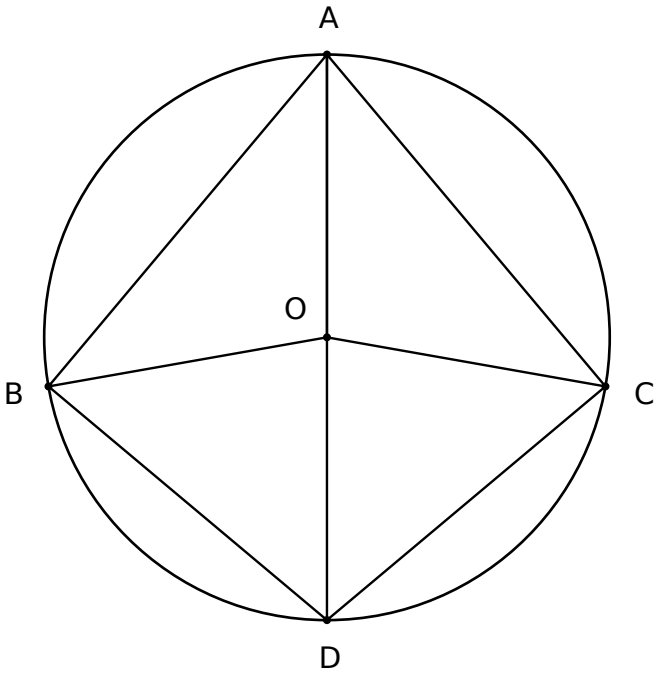
Green dashed lines show added constructions.

Equal Tangents and Congruence

Challenge 1 — Worked Solution

1. OA = OA, OB = OC (radii), and AB = AC (given).
2. Triangles OAB and OAC are congruent by SSS.
3. Thus the smaller central angles AOB and AOC are equal.
4. Angle ADB = half angle AOB and angle ADC = half angle AOC.
5. Therefore angle ADB = angle ADC.

Not accurately drawn



Green dashed lines show added constructions.

Equal Tangents and Congruence

Challenge 2 — Worked Solution

1. OB = OC (radii), so angle OCB = (180 − 2x)/2 = (90 − x)°.

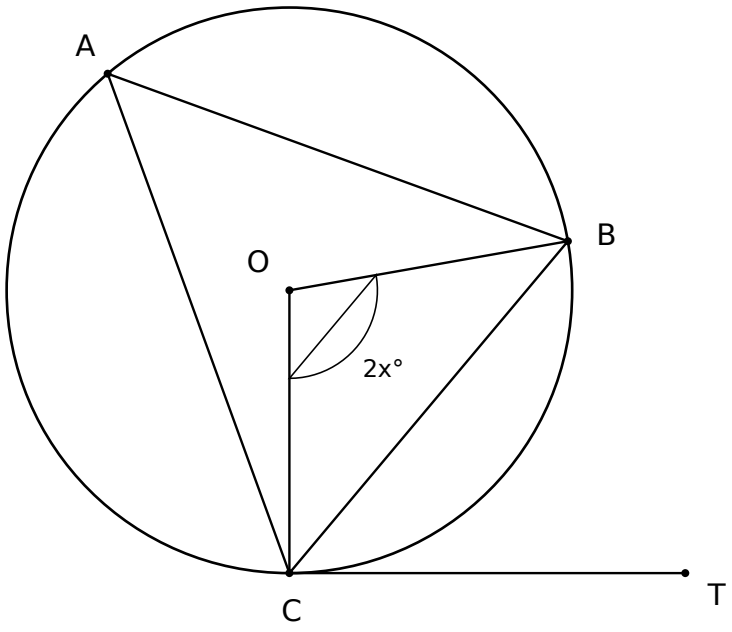
2. OC is perpendicular to tangent CT, so angle OCT = 90°.

3. Angle BCT = 90° − (90 − x)° = x°.

4. By the angle-at-the-centre theorem, angle BAC = (2x)/2 = x°.

5. Hence angle BCT = angle BAC.

Not accurately drawn



Green dashed lines show added constructions.