Coordinate Geometry: Perpendicular Lines & Right-Angled Triangles

Practice Worksheet (Edexcel GCSE 1MA1 Nov 2024 2H Style)

I Do

The point P has coordinates $(-4, 5)$.
The point Q has coordinates $(6, -6)$.
The point R has coordinates $(k, k + 3)$.

Given that angle PRQ is a right angle, find the possible values of k.
You must show all your working.

We Do

The point P has coordinates $(-2, -5)$.
The point Q has coordinates $(5, 4)$.
The point R has coordinates $(k, k + 2)$.

Given that angle PRQ is a right angle, find the possible values of k.
You must show all your working.

You Do

The point P has coordinates $(-5, 1)$.
The point Q has coordinates $(6, -1)$.
The point R has coordinates $(k, k + 2)$.

Given that angle PRQ is a right angle, find the possible values of k.
You must show all your working.

Challenge

The point P has coordinates $(-2, -3)$.
The point Q has coordinates $(2, -5)$.
The point R has coordinates $(k, 2k - 4)$.

Given that angle PRQ is a right angle:

(a) Show that $5k^2 - 5 = 0$.

(b) Hence, find the possible values of k.

Answers & Worked Solutions

- **I Do:**
  1. Gradient PR: $m_1 = \frac{(k+3)-5}{k-(-4)} = \frac{k-2}{k+4}$.
  2. Gradient QR: $m_2 = \frac{(k+3)-(-6)}{k-6} = \frac{k+9}{k-6}$.
  3. Since $\angle PRQ = 90°$, $m_1 \times m_2 = -1 \implies \frac{(k-2)(k+9)}{(k+4)(k-6)} = -1$.
  4. Expand and rearrange: $(k-2)(k+9) = -(k+4)(k-6)$
  $k^2 + 7k - 18 = -(k^2 - 2k - 24) = -k^2 + 2k + 24$.
  5. $2k^2 + 5k - 42 = 0 \implies (2k-7)(k+6) = 0 \implies \mathbf{k = 3.5}$ or $\mathbf{k = -6}$.

- **We Do:**
  1. Gradient PR: $m_1 = \frac{(k+2)-(-5)}{k-(-2)} = \frac{k+7}{k+2}$.
  2. Gradient QR: $m_2 = \frac{(k+2)-4}{k-5} = \frac{k-2}{k-5}$.
  3. Since $\angle PRQ = 90°$, $m_1 \times m_2 = -1 \implies \frac{(k+7)(k-2)}{(k+2)(k-5)} = -1$.
  4. $(k+7)(k-2) = -(k+2)(k-5) \implies k^2 + 5k - 14 = -(k^2 - 3k - 10) = -k^2 + 3k + 10$.
  5. $2k^2 + 2k - 24 = 0 \implies k^2 + k - 12 = 0 \implies (k+4)(k-3) = 0$.
  6. $\mathbf{k = 3}$ or $\mathbf{k = -4}$.

- **You Do:**
  1. Gradient PR: $m_1 = \frac{(k+2)-1}{k-(-5)} = \frac{k+1}{k+5}$.
  2. Gradient QR: $m_2 = \frac{(k+2)-(-1)}{k-6} = \frac{k+3}{k-6}$.
  3. Since $\angle PRQ = 90°$, $m_1 \times m_2 = -1 \implies \frac{(k+1)(k+3)}{(k+5)(k-6)} = -1$.
  4. $(k+1)(k+3) = -(k+5)(k-6) \implies k^2 + 4k + 3 = -(k^2 - k - 30) = -k^2 + k + 30$.
  5. $2k^2 + 3k - 27 = 0 \implies (2k-9)(k+3) = 0$.
  6. $\mathbf{k = 4.5}$ or $\mathbf{k = -3}$.

- **Challenge:**
  1. (a) Gradient PR: $m_1 = \frac{2k-4-(-3)}{k-(-2)} = \frac{2k-1}{k+2}$.
  Gradient QR: $m_2 = \frac{2k-4-(-5)}{k-2} = \frac{2k+1}{k-2}$.
  $m_1 \times m_2 = -1 \implies (2k-1)(2k+1) = -(k+2)(k-2)$.
  $4k^2 - 1 = -(k^2 - 4) = -k^2 + 4 \implies \mathbf{5k^2 - 5 = 0}$ ✓
  2. (b) $5k^2 = 5 \implies k^2 = 1 \implies \mathbf{k = 1}$ or $\mathbf{k = -1}$.