Iterative Sequences Worksheet

Repeated Change and Bouncing Balls

I Do

A ball is thrown upwards and reaches a maximum height.
The ball then falls and bounces repeatedly.

After the nth bounce, the ball reaches a height of h_n.
After the next bounce, the ball reaches a height given by $h_{n+1} = 0.55h_n$.

After the 1st bounce, the ball reaches a height of 8 metres.

What height does the ball reach after the 4th bounce?

We Do

A rubber ball is dropped and bounces repeatedly.
After the nth bounce, the ball reaches a height of h_n cm.
After the next bounce, the ball reaches a height given by $h_{n+1} = 0.65h_n$.

After the 1st bounce, the ball reaches a height of 120 cm.

What height does the ball reach after the 4th bounce?

You Do

A tennis ball is dropped and bounces repeatedly.
After the nth bounce, the ball reaches a height of h_n.
After the next bounce, the ball reaches a height given by $h_{n+1} = 0.8h_n$.

After the 1st bounce, the ball reaches a height of 5 metres.

What height does the ball reach after the 4th bounce?

Challenge

A bouncy ball is dropped from an initial height of 15 metres.
After each bounce, it reaches a height that is 60% of its previous maximum height.
Let h_n be the height after the nth bounce.

(a) Write down an iterative formula for h_{n+1} in terms of h_n.
(b) Find the state the value of h_1.
(c) After which bounce does the ball first reach a maximum height of less than 2 metres?

Answers & Worked Solutions

• **I Do:**
$h_1 = 8$
$h_2 = 0.55 \times 8 = 4.4$
$h_3 = 0.55 \times 4.4 = 2.42$
$h_4 = 0.55 \times 2.42 = 1.331$

Answer: 1.331 ***metres***

• **We Do:**
$h_1 = 120$
$h_2 = 0.65 \times 120 = 78$
$h_3 = 0.65 \times 78 = 50.7$
$h_4 = 0.65 \times 50.7 = 32.955$

Answer: 32.955 ***cm***

• **You Do:**
$h_1 = 5$
$h_2 = 0.8 \times 5 = 4$
$h_3 = 0.8 \times 4 = 3.2$
$h_4 = 0.8 \times 3.2 = 2.56$

Answer: 2.56 ***metres***

• **Challenge:**
(a) 60% ***as a multiplier is*** 0.6***. So,*** $h_{n+1} = 0.6h_n$
(b) h_1 ***is the height after the first bounce:*** $15 \times 0.6 = 9$ ***metres***
(c) Keep iterating until the height is less than 2***:***
$h_1 = 9$
$h_2 = 0.6 \times 9 = 5.4$
$h_3 = 0.6 \times 5.4 = 3.24$
$h_4 = 0.6 \times 3.24 = 1.944$
$1.944 < 2$***, so it happens after the 4th bounce.***

Answer: (a) $h_{n+1} = 0.6h_n$ ***(b)*** $h_1 = 9$ ***(c) 4th bounce***