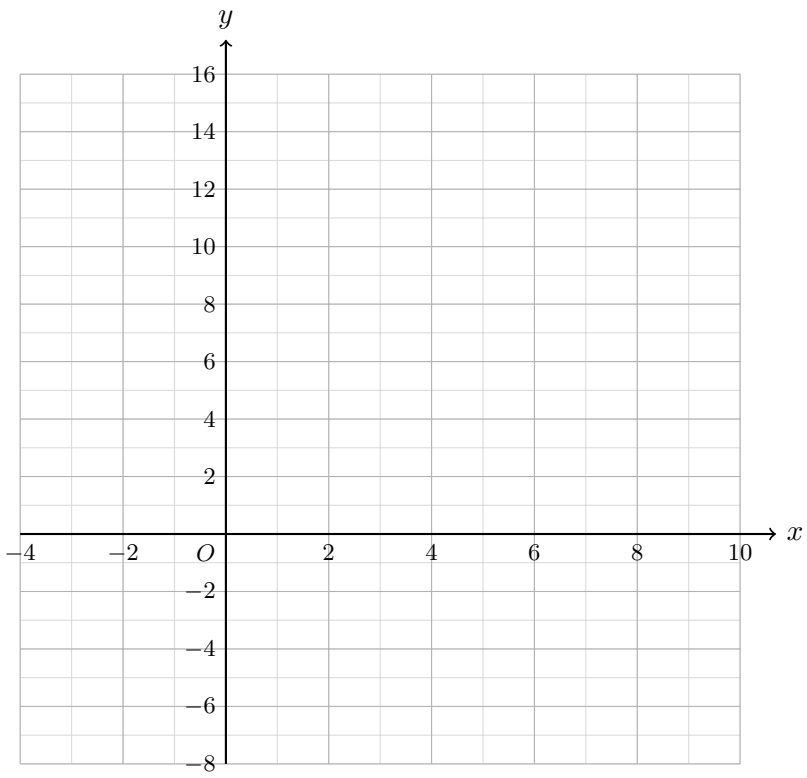
Graphical Inequalities & Integer Points

Practice Worksheet (Problem Solving)

I Do

(a) On the grid, draw the four boundary lines and shade the region that satisfies all four inequalities. Label the region R.

$x > 1$ $y < 11$ $2x - y < 5$ $3x + 2y > 15$



(4 marks)

(b) Find n, the number of points in R with coordinates (4, y) where y is an integer.

$n =$ ..... *(1 mark)*

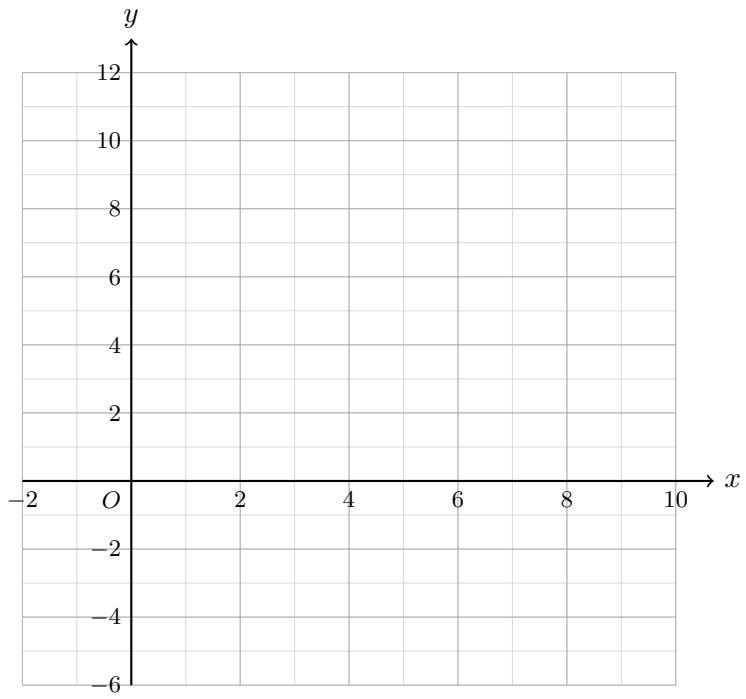
(Total for Question is 5 marks)

We Do

(a) Show, by shading, the region that satisfies all these inequalities.

$$x > 0 \qquad y < 8 \qquad 2x - y < 4 \qquad 3x + 2y > 12$$

Label the region **R**.



(4 marks)

In region **R** there are n points with coordinates of the form $(2, y)$ where y is an integer.
(b) Find the value of n.

$n =$ ..... *(1 mark)*

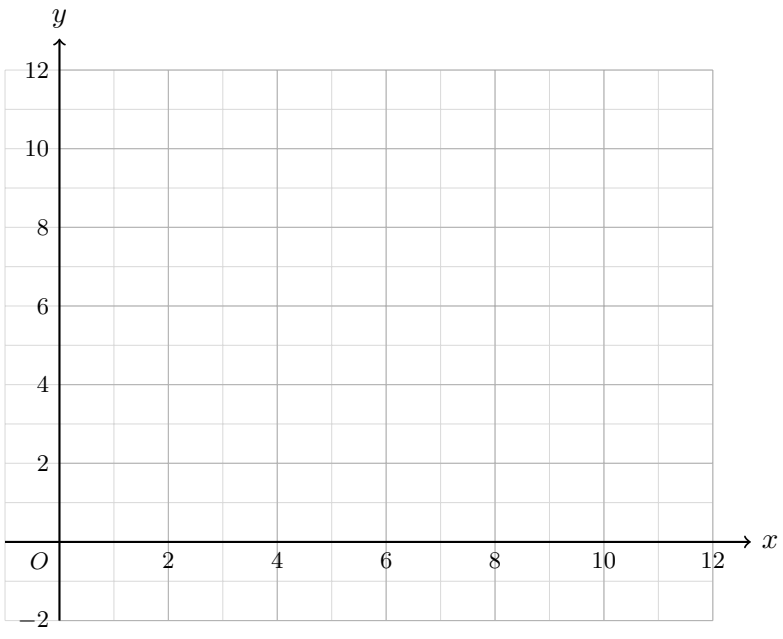
(Total for Question is 5 marks)

You Do 1

(a) Show, by shading, the region that satisfies all these inequalities.

$$x \geq 1 \qquad y \leq 9 \qquad x - y < 2 \qquad x + y > 6$$

Label the region **R**.



(4 marks)

In region **R** there are n points with coordinates of the form $(4, y)$ where y is an integer.
(b) Find the value of n.

$n =$ ..... *(1 mark)*

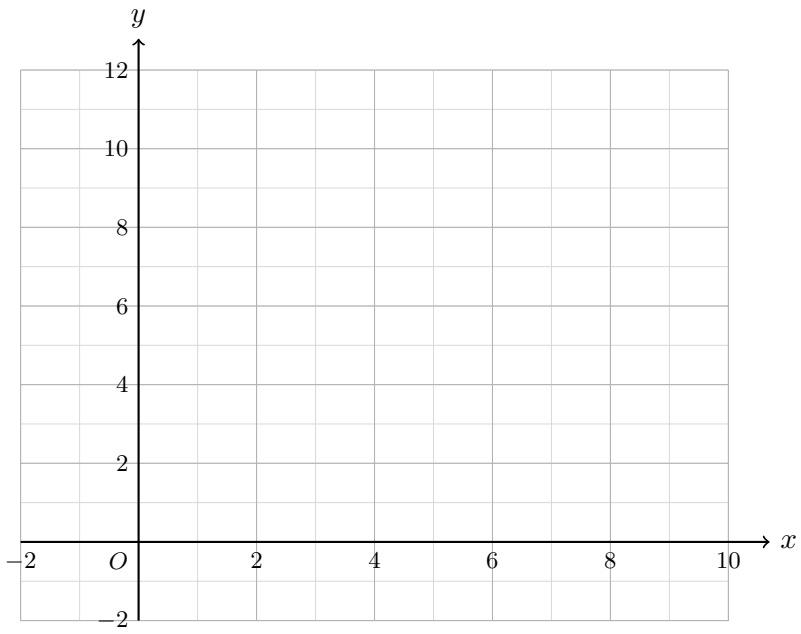
(Total for Question is 5 marks)

You Do 2

(a) Show, by shading, the region that satisfies all these inequalities.

$$y > 0 \qquad x < 8 \qquad y < 2x + 2 \qquad 2x + y > 8$$

Label the region **R**.



(4 marks)

In region **R** there are n points with coordinates of the form $(x, 4)$ where x is an integer.
(b) Find the value of n.

$n =$ ..... *(1 mark)*

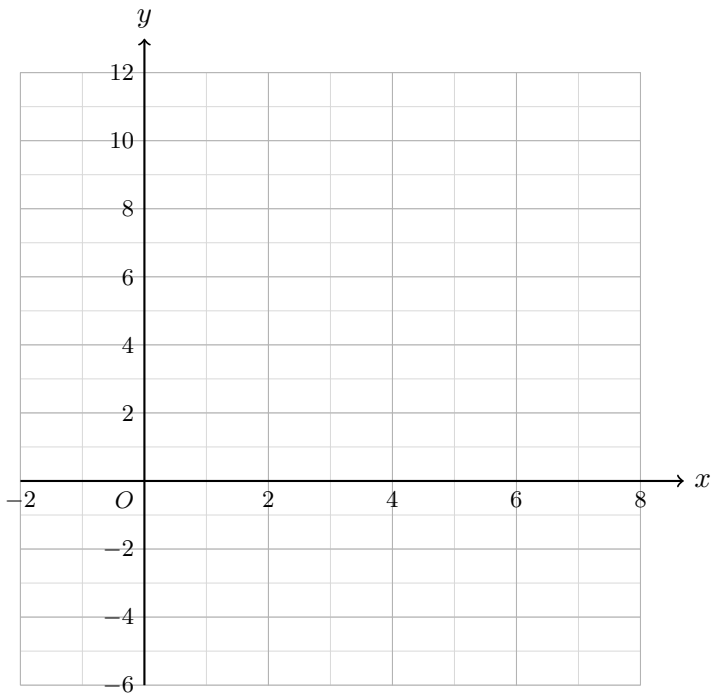
(Total for Question is 5 marks)

You Do 3

(a) Show, by shading, the region that satisfies all these inequalities.

$$x \geq 0 \qquad y \leq 10 \qquad 3x - y \leq 4 \qquad 3x + 4y \geq 12$$

Label the region **R**.



(4 marks)

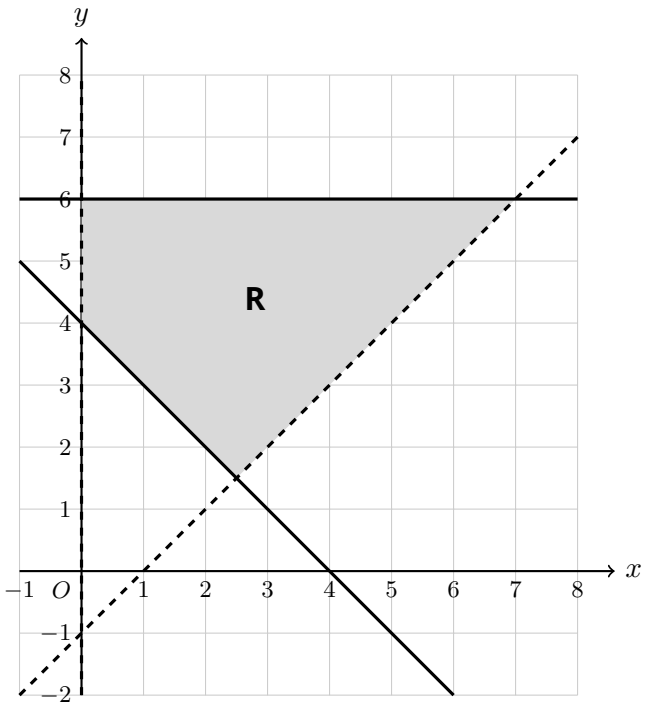
In region **R** there are n points with coordinates of the form $(2, y)$ where y is an integer.
(b) Find the value of n.

$n =$ ..... *(1 mark)*

(Total for Question is 5 marks)

Challenge 1

The shaded region **R** shown on the grid is defined by four inequalities.



(a) Write down the four inequalities that define the shaded region **R**.

.... *(4 marks)*

(b) Find the number of points in region **R** with coordinates of the form $(x, 3)$ where x is an integer.

..... *(1 mark)*

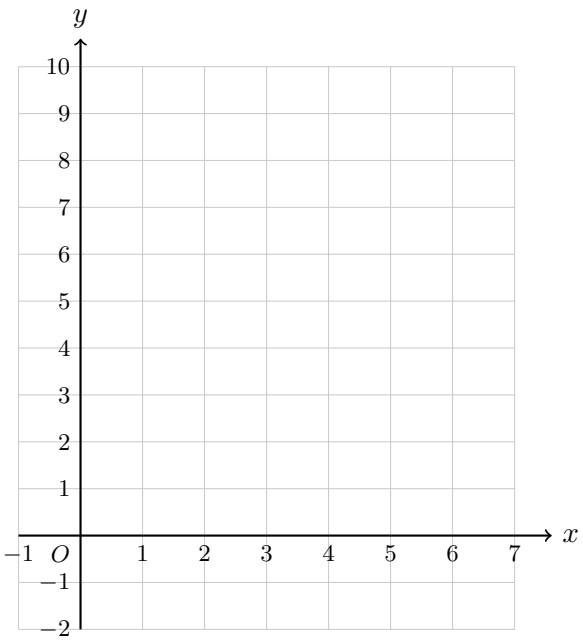
(Total for Question is 5 marks)

Challenge 2

(a) Show, by shading, the region **R** that satisfies all four inequalities.

$$x \geq 0 \qquad y \leq 8 \qquad y \geq 2x - 2 \qquad x + y \geq 4$$

Label the region **R**.



(4 marks)

(b) Calculate the exact area of the region **R**.

.....units^2 *(2 marks)*

(Total for Question is 6 marks)

Extreme Challenge

Region **R** is defined by the four inequalities:

$$x > 0 \qquad y < k \qquad 2x - y < 6 \qquad 5x + 4y > 20$$

where k is a positive integer.

In region **R**, there are exactly 12 points with coordinates of the form $(3, y)$, where y is an integer.

(a) Find the value of k. Show your working clearly.

$k =$ ..... *(3 marks)*

(b) Using your value of k, find the greatest possible integer value of x for a point (x, y) with integer coordinates that lies inside region **R**.
You must show all of your working clearly.

$x =$ ..... *(3 marks)*

(Total for Question is 6 marks)

Worked Answers

I Do

(a) Draw dashed lines x = 1, y = 11, y = 2x − 5 and y = (15 − 3x)/2.

All four lines are dashed because the inequalities are strict.

Shade to the right of x = 1, below y = 11 and above both sloping lines. Label this region R.

(b) Substitute x = 4 into the inequalities:

2(4) − y < 5 gives y > 3; 3(4) + 2y > 15 gives y > 1.5.

Together with y < 11, the strongest bounds are 3 < y < 11.

The integers in 3 < y < 11 are 4, 5, 6, 7, 8, 9 and 10, so n = 7.

We Do

(a) Draw four **dashed** lines: $x = 0$, $y = 8$, $y = 2x - 4$ (through $(0, -4), (2, 0), (6, 8)$), and $3x + 2y = 12$ (through $(0, 6), (4, 0)$).
(b) At $x = 2$: $y < 8$, $y > 2(2) - 4 = 0$, and $6 + 2y > 12 \implies y > 3$.
So $3 < y < 8 \implies y \in \{4, 5, 6, 7\} \implies$ **n = 4**.

You Do 1

(a) Solid lines: $x = 1$ and $y = 9$. Dashed lines: $y = x - 2$ (through $(2, 0), (11, 9)$) and $x + y = 6$ (through $(0, 6), (6, 0)$).
(b) At $x = 4$: $y \leq 9$, $y > 4 - 2 = 2$, and $y > 6 - 4 = 2$.
So $2 < y \leq 9 \implies y \in \{3, 4, 5, 6, 7, 8, 9\} \implies$ **n = 7**.

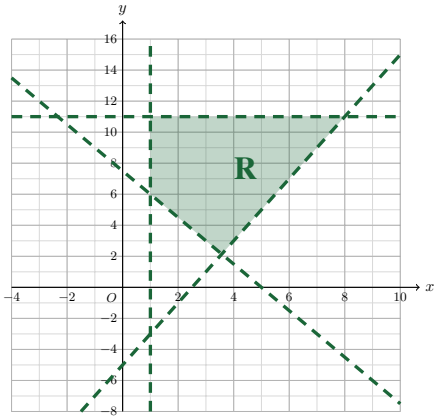
You Do 2

(a) Four **dashed** lines: $y = 0$, $x = 8$, $y = 2x + 2$ (through $(-1, 0), (0, 2), (5, 12)$), and $2x + y = 8$ (through $(0, 8), (4, 0)$).
(b) At $y = 4$: $x < 8$, $4 < 2x + 2 \implies x > 1$, and $2x + 4 > 8 \implies x > 2$.
So $2 < x < 8 \implies x \in \{3, 4, 5, 6, 7\} \implies$ **n = 5**.

You Do 3

(a) Four **solid** lines: $x = 0$, $y = 10$, $y = 3x - 4$ (through $(0, -4), (2, 2), (4, 8)$), and $3x + 4y = 12$ (through $(0, 3), (4, 0)$).
(b) At $x = 2$: $y \leq 10$, $y \geq 3(2) - 4 = 2$, and $6 + 4y \geq 12 \implies y \geq 1.5$.
So $2 \leq y \leq 10 \implies y \in \{2, 3, 4, 5, 6, 7, 8, 9, 10\} \implies$ **n = 9**.

I Do (a): Shaded region R



Challenge 1

(a) Identify each boundary line and whether it is solid or dashed:
1. Left edge: dashed line $x = 0$, region is to the right $\implies$ **x** > 0.
2. Top edge: solid line $y = 6$, region is below $\implies$ **y** ≤ 6.
3. Bottom-right edge: dashed line through $(1, 0)$ and $(7, 6)$ has equation $y = x - 1$, region is above $\implies$ **y** $>$ **x** $-$ **1** *(or* $x - y < 1$*)*.
4. Bottom-left edge: solid line through $(0, 4)$ and $(4, 0)$ has equation $x + y = 4$, region is above $\implies$ **x** $+$ **y** $\geq$ **4**.

(b) Substitute $y = 3$: $x > 0$, $3 > x - 1 \implies x < 4$, and $x + 3 \geq 4 \implies x \geq 1$.
So $1 \leq x < 4 \implies x \in \{1, 2, 3\} \implies$ **3 points**.

Challenge 2

(a) Draw four **solid** lines: $x = 0$, $y = 8$, $y = 2x - 2$ (through $(0, -2), (2, 2), (5, 8)$), and $x + y = 4$ (through $(0, 4), (2, 2), (4, 0)$).
Region R is the quadrilateral with vertices $(0, 8)$, $(5, 8)$, $(2, 2)$, and $(0, 4)$.

(b) Enclose **R** in a 5×6 rectangle from $x = 0$ to 5 and $y = 2$ to 8 (area $= 30$):
- Subtract bottom-left triangle with vertices $(0, 4), (0, 2), (2, 2)$: Area $= \frac{1}{2} \times 2 \times 2 = 2$.
- Subtract bottom-right triangle with vertices $(2, 2), (5, 2), (5, 8)$: Area $= \frac{1}{2} \times 3 \times 6 = 9$.

Area of **R** $= 30 - 2 - 9 =$ **19** units^2.

Extreme Challenge

(a) Substitute $x = 3$ into the inequalities:
- $2(3) - y < 6 \implies y > 0$
- $5(3) + 4y > 20 \implies 4y > 5 \implies y > 1.25$
- $y < k$.

So y must satisfy $1.25 < y < k$. The smallest integer value of y is 2.
For there to be 12 integer values of y, they must be $2, 3, 4, \ldots, 13$.
Since $y < k$ is a strict inequality, **k** $=$ **14**.

(b) Since $y < 14$ and y is an integer, the largest possible value of y is 13.
To maximise x, substitute $y = 13$ into $2x - y < 6$:
$2x - 13 < 6 \implies 2x < 19 \implies x < 9.5$.
Testing $(9, 13)$ in all four inequalities: $9 > 0$, $13 < 14$, $2(9) - 13 = 5 < 6$, and $5(9) + 4(13) = 97 > 20$.
Therefore, the greatest possible integer value is **x** $=$ **9**.