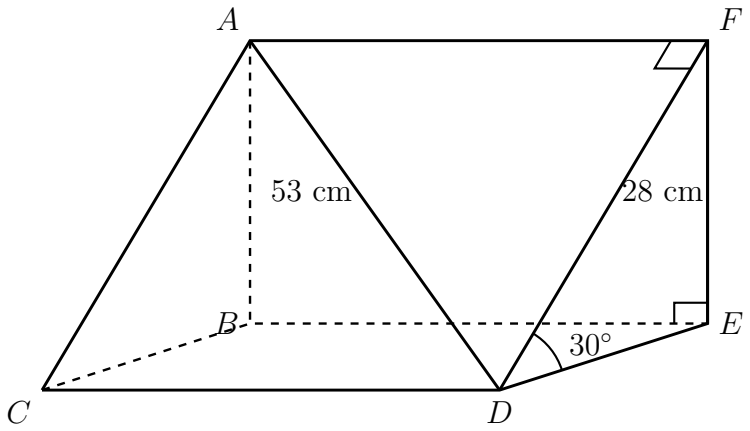
3D Geometry: Volumes of Triangular Prisms

Practice Worksheet (Nov 2023 Paper 1H Style)

I Do

Here is a triangular prism $ABCDEF$.

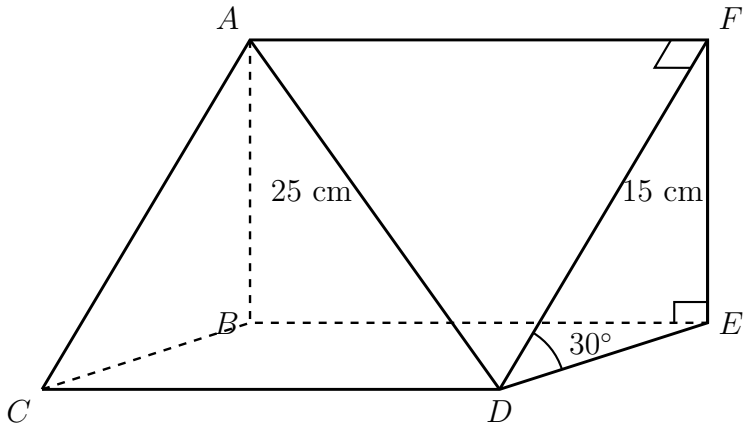


$AD = 53$ cm
$DF = 28$ cm
Angle $FDE = 30°$

Work out the volume of the triangular prism.
Give your answer correct to the nearest whole number.

We Do

Here is a triangular prism $ABCDEF$.

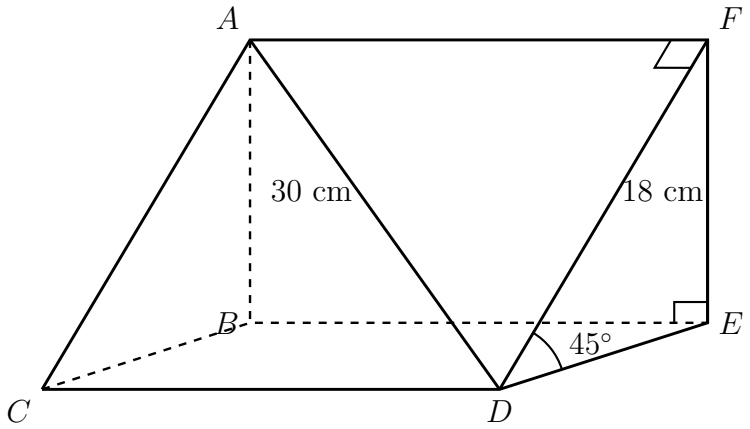


$AD = 25$ cm
$DF = 15$ cm
Angle $FDE = 30°$

Work out the volume of the triangular prism.
Give your answer correct to the nearest whole number.

You Do

Here is a triangular prism $ABCDEF$.

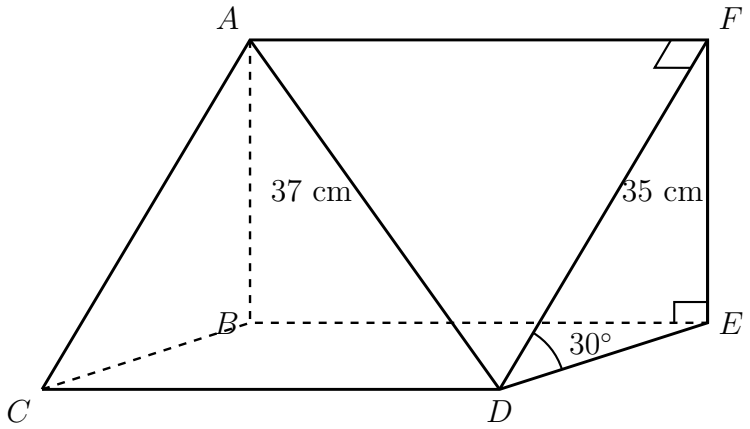


$AD = 30$ cm
$DF = 18$ cm
Angle $FDE = 45°$

Work out the volume of the triangular prism.
Give your answer correct to the nearest whole number.

Challenge

Here is a triangular prism $ABCDEF$.



$AD = 37$ cm
$DF = 35$ cm
Angle $FDE = 30°$

(a) Work out the volume of the triangular prism.
Give your answer correct to the nearest whole number.

(b) Work out the total surface area of the triangular prism.
Give your answer correct to the nearest whole number.

Answers & Worked Solutions

- **I Do:**
  1. Prism length AF using Pythagoras in right-angled $\triangle AFD$:
  $$AF = \sqrt{AD^2 - DF^2} = \sqrt{53^2 - 28^2} = \sqrt{2809 - 784} = \sqrt{2025} = 45 \text{ cm}$$
  2. Side lengths of right-angled $\triangle DEF$:
  $$FE = 28 \sin 30^\circ = 14 \text{ cm}, \quad DE = 28 \cos 30^\circ = 14\sqrt{3} \text{ cm} \approx 24.2487 \text{ cm}$$
  3. Cross-sectional Area of $\triangle DEF$:
  $$\text{Area} = \frac{1}{2} \times 14\sqrt{3} \times 14 = 98\sqrt{3} \approx 169.7409 \text{ cm}^2$$
  4. Volume $= 98\sqrt{3} \times 45 = 4410\sqrt{3} \approx 7638.344 \implies \mathbf{7638}$ cm$^\mathbf{3}$.

- **We Do:**
  1. $AF = \sqrt{25^2 - 15^2} = \sqrt{400} = 20$ cm.
  2. $FE = 15 \sin 30^\circ = 7.5$ cm, $\quad DE = 15 \cos 30^\circ = 7.5\sqrt{3}$ cm.
  3. Area $\triangle DEF = \frac{1}{2} \times 7.5\sqrt{3} \times 7.5 = 28.125\sqrt{3} \approx 48.7139$ cm^2.
  4. Volume $= 28.125\sqrt{3} \times 20 = 562.5\sqrt{3} \approx 974.278 \implies \mathbf{974}$ cm$^\mathbf{3}$.

- **You Do:**
  1. $AF = \sqrt{30^2 - 18^2} = \sqrt{576} = 24$ cm.
  2. $FE = 18 \sin 45^\circ = 9\sqrt{2}$ cm, $\quad DE = 18 \cos 45^\circ = 9\sqrt{2}$ cm.
  3. Area $\triangle DEF = \frac{1}{2} \times 9\sqrt{2} \times 9\sqrt{2} = 81$ cm^2.
  4. Volume $= 81 \times 24 = \mathbf{1944}$ cm$^\mathbf{3}$.

- **Challenge:**
  1. $AF = \sqrt{37^2 - 35^2} = \sqrt{144} = 12$ cm.
  2. $FE = 35 \sin 30^\circ = 17.5$ cm, $\quad DE = 35 \cos 30^\circ = 17.5\sqrt{3}$ cm ≈ 30.3109 cm.
  3. (a) Volume $= \left(\frac{1}{2} \times 17.5\sqrt{3} \times 17.5\right) \times 12 = 1837.5\sqrt{3} \approx 3182.64 \implies \mathbf{3183}$ cm$^\mathbf{3}$.
  4. (b) Total Surface Area $= 2(\text{Area } \triangle DEF) + (\text{Perimeter } \triangle DEF \times AF)$
  Perimeter $\triangle DEF = 17.5 + 17.5\sqrt{3} + 35 = 52.5 + 17.5\sqrt{3} \approx 82.8109$ cm.
  Total Surface Area $= 2(153.125\sqrt{3}) + (52.5 + 17.5\sqrt{3}) \times 12$
  $= 306.25\sqrt{3} + 630 + 210\sqrt{3} = 630 + 516.25\sqrt{3} \approx 1524.18 \implies \mathbf{1524}$ cm$^\mathbf{2}$.