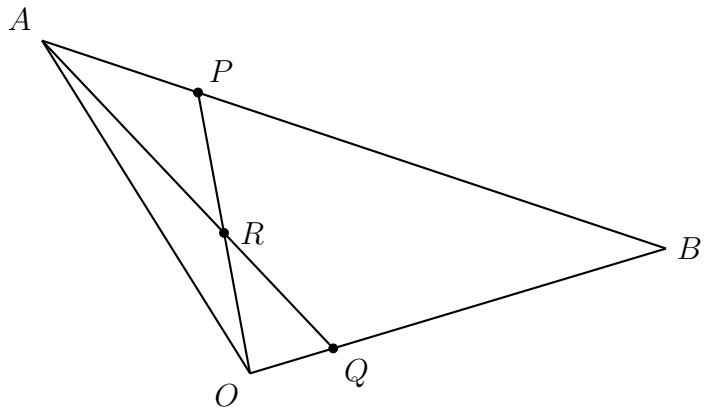
Vectors: Line Intersections in Triangles

Practice Worksheet (Nov 2024 Paper 1H Style)

I Do



OAB is a triangle with $\overrightarrow{OA} = 10\mathbf{a}$ and $\overrightarrow{OB} = 10\mathbf{b}$.
ARQ and ORP are straight lines.
$\overrightarrow{AP} = \frac{1}{4}\overrightarrow{AB}$ and $\overrightarrow{OQ} = \frac{1}{5}\overrightarrow{OB}$.

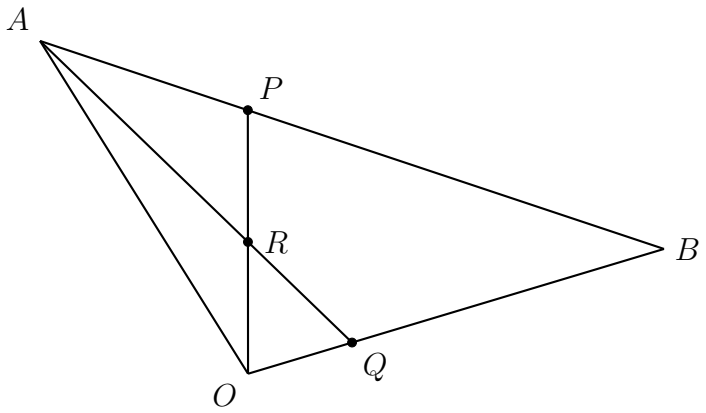
Write the following vectors in terms of **a** and **b**. Simplify your answers.

(i) $\overrightarrow{AQ}$

(ii) $\overrightarrow{OP}$

(iii) $\overrightarrow{OR}$

We Do



OAB is a triangle with $\overrightarrow{OA} = 12\mathbf{a}$ and $\overrightarrow{OB} = 12\mathbf{b}$.
ARQ and ORP are straight lines.
$\overrightarrow{AP} = \frac{1}{3}\overrightarrow{AB}$ and $\overrightarrow{OQ} = \frac{1}{4}\overrightarrow{OB}$.

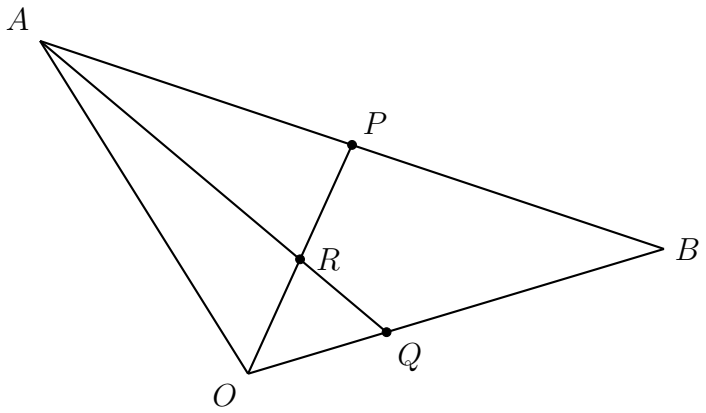
Write the following vectors in terms of **a** and **b**. Simplify your answers.

(i) $\overrightarrow{AQ}$

(ii) $\overrightarrow{OP}$

(iii) $\overrightarrow{OR}$

You Do



OAB is a triangle with $\overrightarrow{OA} = 6\mathbf{a}$ and $\overrightarrow{OB} = 6\mathbf{b}$.
ARQ and ORP are straight lines.
$\overrightarrow{AP} = \frac{1}{2}\overrightarrow{AB}$ and $\overrightarrow{OQ} = \frac{1}{3}\overrightarrow{OB}$.

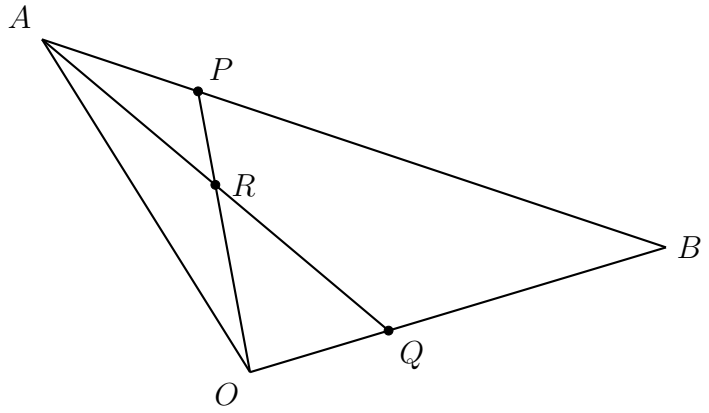
Write the following vectors in terms of **a** and **b**. Simplify your answers.

(i) $\overrightarrow{AQ}$

(ii) $\overrightarrow{OP}$

(iii) $\overrightarrow{OR}$

Challenge



OAB is a triangle with $\overrightarrow{OA} = 12\mathbf{a}$ and $\overrightarrow{OB} = 12\mathbf{b}$.
ARQ and ORP are straight lines.
$\overrightarrow{AP} = \frac{1}{4}\overrightarrow{AB}$ and $\overrightarrow{OQ} = \frac{1}{3}\overrightarrow{OB}$.

Write the following vectors in terms of **a** and **b**. Simplify your answers.

(i) $\overrightarrow{AQ}$

(ii) $\overrightarrow{OP}$

(iii) $\overrightarrow{OR}$

Answers

- **I Do:**

$$\text{(i) } \overrightarrow{AQ} = \overrightarrow{AO} + \overrightarrow{OQ} = -10\mathbf{a} + \frac{1}{5}(10\mathbf{b}) = \mathbf{-10a + 2b}$$

$$\text{(ii) } \overrightarrow{AB} = -10\mathbf{a} + 10\mathbf{b} \implies \overrightarrow{AP} = \frac{1}{4}(-10\mathbf{a} + 10\mathbf{b}) = -\frac{5}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}$$

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = 10\mathbf{a} - \frac{5}{2}\mathbf{a} + \frac{5}{2}\mathbf{b} = \mathbf{\frac{15}{2}a} + \mathbf{\frac{5}{2}b} \quad (\text{or } 7.5\mathbf{a} + 2.5\mathbf{b})$$

$$\text{(iii) } \overrightarrow{OR} = k\overrightarrow{OP} = \frac{15}{2}k\mathbf{a} + \frac{5}{2}k\mathbf{b}$$

$$\overrightarrow{OR} = \overrightarrow{OA} + m\overrightarrow{AQ} = 10\mathbf{a} + m(-10\mathbf{a} + 2\mathbf{b}) = 10(1-m)\mathbf{a} + 2m\mathbf{b}$$

$$\text{Equating coeffs: } \frac{5}{2}k = 2m \implies m = \frac{5}{4}k$$

$$\frac{15}{2}k = 10\left(1 - \frac{5}{4}k\right) = 10 - \frac{25}{2}k \implies 20k = 10 \implies k = \frac{1}{2}$$

$$\overrightarrow{OR} = \frac{1}{2}\left(\frac{15}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}\right) = \mathbf{\frac{15}{4}a} + \mathbf{\frac{5}{4}b} \quad (\text{or } 3.75\mathbf{a} + 1.25\mathbf{b})$$

- **We Do:**

$$\text{(i) } \overrightarrow{AQ} = -12\mathbf{a} + 3\mathbf{b} = \mathbf{-12a + 3b}$$

$$\text{(ii) } \overrightarrow{OP} = 12\mathbf{a} + \frac{1}{3}(-12\mathbf{a} + 12\mathbf{b}) = \mathbf{8a + 4b}$$

$$\text{(iii) } \overrightarrow{OR} = k(8\mathbf{a} + 4\mathbf{b}) = 12(1-m)\mathbf{a} + 3m\mathbf{b} \implies 4k = 3m \implies m = \frac{4}{3}k$$

$$8k = 12\left(1 - \frac{4}{3}k\right) = 12 - 16k \implies 24k = 12 \implies k = \frac{1}{2}$$

$$\overrightarrow{OR} = \frac{1}{2}(8\mathbf{a} + 4\mathbf{b}) = \mathbf{4a + 2b}$$

- **You Do:**

$$\text{(i) } \overrightarrow{AQ} = -6\mathbf{a} + 2\mathbf{b} = \mathbf{-6a + 2b}$$

$$\text{(ii) } \overrightarrow{OP} = 6\mathbf{a} + \frac{1}{2}(-6\mathbf{a} + 6\mathbf{b}) = \mathbf{3a + 3b}$$

$$\text{(iii) } \overrightarrow{OR} = k(3\mathbf{a} + 3\mathbf{b}) = 6(1-m)\mathbf{a} + 2m\mathbf{b} \implies 3k = 2m \implies m = \frac{3}{2}k$$

$$3k = 6\left(1 - \frac{3}{2}k\right) = 6 - 9k \implies 12k = 6 \implies k = \frac{1}{2}$$

$$\overrightarrow{OR} = \frac{1}{2}(3\mathbf{a} + 3\mathbf{b}) = \mathbf{\frac{3}{2}a} + \mathbf{\frac{3}{2}b} \quad (\text{or } 1.5\mathbf{a} + 1.5\mathbf{b})$$

- **Challenge:**

$$\text{(i) } \overrightarrow{AQ} = -12\mathbf{a} + 4\mathbf{b} = \mathbf{-12a + 4b}$$

$$\text{(ii) } \overrightarrow{OP} = 12\mathbf{a} + \frac{1}{4}(-12\mathbf{a} + 12\mathbf{b}) = \mathbf{9a + 3b}$$

$$\text{(iii) } \overrightarrow{OR} = k(9\mathbf{a} + 3\mathbf{b}) = 12(1-m)\mathbf{a} + 4m\mathbf{b} \implies 3k = 4m \implies m = \frac{3}{4}k$$

$$9k = 12\left(1 - \frac{3}{4}k\right) = 12 - 9k \implies 18k = 12 \implies k = \frac{2}{3}$$

$$\overrightarrow{OR} = \frac{2}{3}(9\mathbf{a} + 3\mathbf{b}) = \mathbf{6a + 2b}$$