Probability: Dependent Events (Without Replacement)

Practice Worksheet (Jun 2023 Paper 2H Style)

I Do

There are 12 counters in a bag. 3 of the counters are red. 9 of the counters are green.

Ameya, Jack and Ella each take at random one counter from the bag. Work out the probability that at least one red counter is still in the bag.

We Do

There are 10 counters in a bag. 2 of the counters are blue. 8 of the counters are yellow.

Liam and Noah each take at random one counter from the bag. Work out the probability that at least one blue counter is still in the bag.

You Do

There are 15 counters in a bag. 4 of the counters are black. 11 of the counters are white.

Four friends each take at random one counter from the bag. Work out the probability that at least one black counter is still in the bag.

Challenge

There are n counters in a bag. 3 of the counters are red and the rest are green.

Two counters are taken at random from the bag without replacement. The probability that both counters are red is $\frac{1}{15}$.

Work out the total number of counters, n, in the bag. Show clear algebraic working.

Answers

- **I Do:**

$$\text{P(0 red left)} = \text{P(Red, Red, Red)} = \frac{3}{12} \times \frac{2}{11} \times \frac{1}{10} = \frac{6}{1320} = \frac{1}{220}$$

$$\text{P(at least 1 red left)} = 1 - \frac{1}{220} = \mathbf{\frac{219}{220}}$$

- **We Do:**

$$\text{P(0 blue left)} = \text{P(Blue, Blue)} = \frac{2}{10} \times \frac{1}{9} = \frac{2}{90} = \frac{1}{45}$$

$$\text{P(at least 1 blue left)} = 1 - \frac{1}{45} = \mathbf{\frac{44}{45}}$$

- **You Do:**

$$\text{P(0 black left)} = \text{P(Black, Black, Black, Black)} = \frac{4}{15} \times \frac{3}{14} \times \frac{2}{13} \times \frac{1}{12} = \frac{24}{32760} = \frac{1}{1365}$$

$$\text{P(at least 1 black left)} = 1 - \frac{1}{1365} = \mathbf{\frac{1364}{1365}}$$

- **Challenge:**

$$\text{P(Red, Red)} = \frac{3}{n} \times \frac{2}{n-1} = \frac{6}{n(n-1)}$$

$$\frac{6}{n^2 - n} = \frac{1}{15} \implies n^2 - n = 90 \implies n^2 - n - 90 = 0$$

$$(n - 10)(n + 9) = 0 \implies n = 10 \text{ (since } n > 0)$$

$$\mathbf{n = 10}$$