Non-Linear Simultaneous Equations

Practice Worksheet (Jun 2023 Paper 2H Style)

I Do

Solve the simultaneous equations

$$2x^2 + 3y^2 = 11$$
$$x = 3y - 1$$

Show clear algebraic working.

We Do

Solve the simultaneous equations

$$3x^2 + 2y^2 = 62$$
$$y = 3x - 1$$

Show clear algebraic working.

You Do

Solve the simultaneous equations

$$2x^2 + 3y^2 = 29$$
$$y = 2x + 1$$

Show clear algebraic working.

Challenge

Solve the simultaneous equations

$$x^2 + xy - y^2 = 11$$
$$x - 2y = 1$$

(a) Show clear algebraic working to find the two pairs of solutions for x and y.

(b) Calculate the exact distance between the two points representing these solutions on a coordinate grid. Give your answer as a simplified surd.

Answers

- **I Do:**
  1. Substitute $x = 3y - 1$ into $2x^2 + 3y^2 = 11$:

$$2(3y-1)^2 + 3y^2 = 11 \implies 2(9y^2 - 6y + 1) + 3y^2 = 11$$

  2. Expand and simplify: $18y^2 - 12y + 2 + 3y^2 = 11 \implies 21y^2 - 12y - 9 = 0$.
  3. Divide by 3: $7y^2 - 4y - 3 = 0 \implies (7y+3)(y-1) = 0$.
  4. Solutions for y: $y = 1$ or $y = -\frac{3}{7}$.
  5. Calculate x:

$$y = 1 \implies x = 3(1) - 1 = 2$$

$$y = -\frac{3}{7} \implies x = 3\left(-\frac{3}{7}\right) - 1 = -\frac{16}{7}$$

  Answer: $\mathbf{x = 2, y = 1}$ and $\mathbf{x = -\frac{16}{7}, y = -\frac{3}{7}}$

- **We Do:**
  1. Substitute $y = 3x - 1$ into $3x^2 + 2y^2 = 62$:

$$3x^2 + 2(3x-1)^2 = 62 \implies 3x^2 + 2(9x^2 - 6x + 1) = 62$$

  2. Simplify: $21x^2 - 12x - 60 = 0 \implies 7x^2 - 4x - 20 = 0$.
  3. Factorise: $(7x+10)(x-2) = 0 \implies x = 2$ or $x = -\frac{10}{7}$.
  4. Calculate y:

$$x = 2 \implies y = 3(2) - 1 = 5$$

$$x = -\frac{10}{7} \implies y = 3\left(-\frac{10}{7}\right) - 1 = -\frac{37}{7}$$

  Answer: $\mathbf{x = 2, y = 5}$ and $\mathbf{x = -\frac{10}{7}, y = -\frac{37}{7}}$

- **You Do:**
  1. Substitute $y = 2x + 1$ into $2x^2 + 3y^2 = 29$:

$$2x^2 + 3(2x+1)^2 = 29 \implies 2x^2 + 3(4x^2 + 4x + 1) = 29$$

  2. Simplify: $14x^2 + 12x - 26 = 0 \implies 7x^2 + 6x - 13 = 0$.
  3. Factorise: $(7x+13)(x-1) = 0 \implies x = 1$ or $x = -\frac{13}{7}$.
  4. Calculate y:

$$x = 1 \implies y = 2(1) + 1 = 3$$

$$x = -\frac{13}{7} \implies y = 2\left(-\frac{13}{7}\right) + 1 = -\frac{19}{7}$$

  Answer: $\mathbf{x = 1, y = 3}$ and $\mathbf{x = -\frac{13}{7}, y = -\frac{19}{7}}$

- **Challenge:**
  1. Rearrange $x - 2y = 1 \implies x = 2y + 1$.
  2. Substitute into $x^2 + xy - y^2 = 11$:

$$(2y+1)^2 + (2y+1)y - y^2 = 11 \implies (4y^2 + 4y + 1) + (2y^2 + y) - y^2 = 11$$

  3. Simplify: $5y^2 + 5y - 10 = 0 \implies y^2 + y - 2 = 0 \implies (y+2)(y-1) = 0$.
  4. (a) $y = 1 \implies x = 3$ and $y = -2 \implies x = -3$.
  Solutions: $(3, 1)$ and $(-3, -2)$.
  5. (b) Distance $= \sqrt{(3-(-3))^2 + (1-(-2))^2} = \sqrt{6^2 + 3^2} = \sqrt{45} = \mathbf{3\sqrt{5}}$.