I Do

The diagram shows a square-based pyramid *ABCDE*.

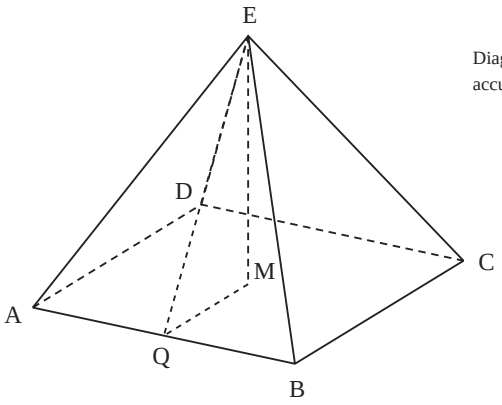


Diagram NOT accurately drawn

$EA = EB = EC = ED$.

M is the centre of the horizontal square base $ABCD$.

Q is the midpoint of AB.

Angle $EQM = 80°$ $EA : AB = n : 1$.

Find the value of n.

Give your answer correct to 3 significant figures.

We Do

The diagram shows a square-based pyramid *ABCDE*.

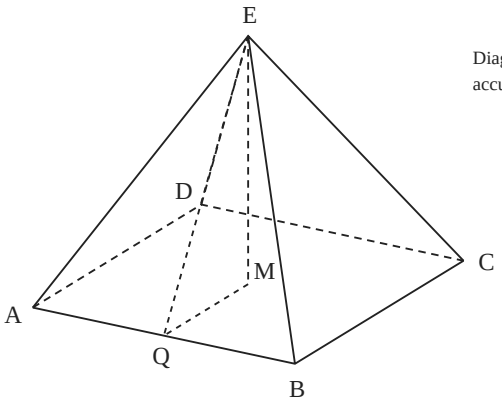


Diagram NOT accurately drawn

$EA = EB = EC = ED$.

M is the centre of the horizontal square base $ABCD$.

Q is the midpoint of AB.

Angle $EQM = 70°$ $EA : AB = n : 1$.

Find the value of n.

Give your answer correct to 3 significant figures.

You Do

The diagram shows a square-based pyramid *ABCDE*.

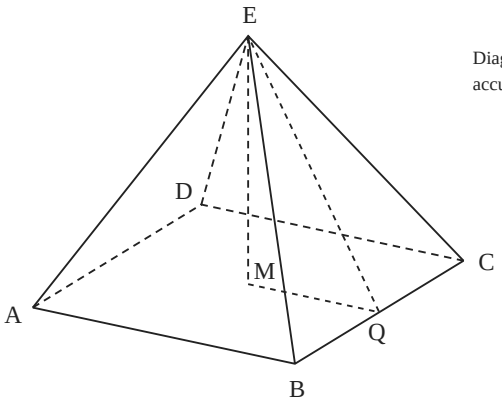


Diagram NOT accurately drawn

$EA = EB = EC = ED$.

M is the centre of the horizontal square base $ABCD$.

Q is the midpoint of BC.

Angle $EQM = 65°$ $EA : BC = n : 1$.

Find the value of n.

Give your answer correct to 3 significant figures.

Challenge

The diagram shows a regular square-based pyramid *ABCDE*.

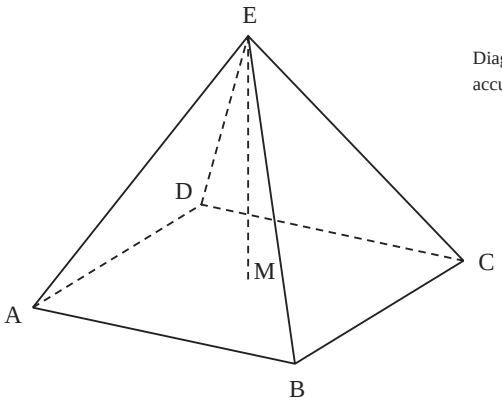


Diagram NOT accurately drawn

EA = *EB* = *EC* = *ED*.

M is the centre of the horizontal square base *ABCD*.

The ratio of *EA* : *AB* = 2 : 1.

The volume of the pyramid is 400 cm^3.

Calculate the total surface area of the pyramid.

Give your answer correct to 3 significant figures.

Answers & Worked Solutions

- **I Do:**
  1. Let the side of the square base $AB = 2$. Then $QA = 1$ and $MQ = 1$.
  2. In right-angled $\triangle EMQ$, $\cos(\angle EQM) = \frac{MQ}{EQ}$.
  $\cos(80°) = \frac{1}{EQ} \implies EQ = \frac{1}{\cos(80°)} = 5.75877\ldots$
  3. In right-angled $\triangle EQA$ (slant face), use Pythagoras' theorem to find EA:
  $EA^2 = EQ^2 + QA^2 = (5.75877)^2 + 1^2 = 33.1634 + 1 = 34.1634$
  $EA = \sqrt{34.1634} = 5.8449\ldots$
  4. Since $EA : AB = n : 1$, and we let $AB = 2$:
  $n = \frac{EA}{AB} = \frac{5.8449}{2} = 2.92247\ldots$
  5. Correct to 3 significant figures: $\mathbf{n = 2.92}$.

- **We Do:**
  1. Let $AB = 2 \implies MQ = 1$ and $QA = 1$.
  2. In $\triangle EMQ$: $\cos(70°) = \frac{1}{EQ} \implies EQ = \frac{1}{\cos(70°)} = 2.9238\ldots$
  3. In $\triangle EQA$: $EA^2 = EQ^2 + 1^2 = 8.5486 + 1 = 9.5486 \implies EA = \sqrt{9.5486} = 3.0900\ldots$
  4. $n = \frac{EA}{AB} = \frac{3.0900}{2} = 1.5450\ldots$
  5. Correct to 3 significant figures: $\mathbf{n = 1.55}$.

- **You Do:**
  (Note: $BC = AB$. Q on BC is symmetrical to Q on AB).
  1. Let $BC = 2 \implies MQ = 1$ and $QC = 1$.
  2. In $\triangle EMQ$: $\cos(65°) = \frac{1}{EQ} \implies EQ = \frac{1}{\cos(65°)} = 2.3662\ldots$
  3. In $\triangle EQC$: $EC^2 = EQ^2 + 1^2 = 5.5989 + 1 = 6.5989 \implies EC = \sqrt{6.5989} = 2.5688\ldots$
  4. Since $EA = EC$, $n = \frac{EA}{BC} = \frac{EC}{BC} = \frac{2.5688}{2} = 1.2844\ldots$
  5. Correct to 3 significant figures: $\mathbf{n = 1.28}$.

- **Challenge:**
  1. Let side $AB = 2x$. Then $EA = 4x$ (since $EA : AB = 2 : 1$). Base Area $= (2x)^2 = 4x^2$.
  2. Q is midpoint of $AB \implies QA = x$ and $MQ = x$.
  3. In $\triangle EQA$: $EQ^2 = EA^2 - QA^2 = (4x)^2 - x^2 = 16x^2 - x^2 = 15x^2 \implies EQ = x\sqrt{15}$.
  4. In $\triangle EMQ$: height $EM^2 = EQ^2 - MQ^2 = 15x^2 - x^2 = 14x^2 \implies EM = x\sqrt{14}$.
  5. Volume $= \frac{1}{3} \times$ Base Area $\times$ Height $= \frac{1}{3}(4x^2)(x\sqrt{14}) = \frac{4\sqrt{14}}{3}x^3 = 400$.
  $x^3 = \frac{1200}{4\sqrt{14}} = \frac{300}{\sqrt{14}} = 80.178\cdots \implies x = 4.312\ldots$ cm.
  6. Total Surface Area = Base Area $+ 4 \times$ (Area of slant $\triangle EAB$)
  $\text{SA} = 4x^2 + 4\left(\frac{1}{2} \cdot 2x \cdot EQ\right) = 4x^2 + 4x(x\sqrt{15}) = 4x^2(1 + \sqrt{15})$.
  Substitute $x = 4.312$: $\text{SA} = 4(4.312)^2(1 + \sqrt{15}) = 4(18.59)(4.8729) = 362.43$ cm^2.
  7. Correct to 3 significant figures: $\mathbf{362}$ cm$^\mathbf{2}$.