Area of Circular Paths

Practice Worksheet (Problem Solving)

I Do

Sophie is building a curved garden path. The path, shown shaded in the diagram, is the region between two semicircles.

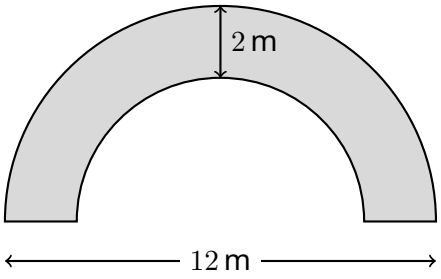


Diagram **NOT** accurately drawn

The diameter of the outer semicircle is 12 m.
The path has a constant width of 2 m.

Calculate the area of the path.
Give your answer as a multiple of π.

.....m^2

(Total for Question is 3 marks)

We Do

Ben is laying a circular path all the way around a circular pond. The path, shown shaded in the diagram, has a constant width of 2 m.

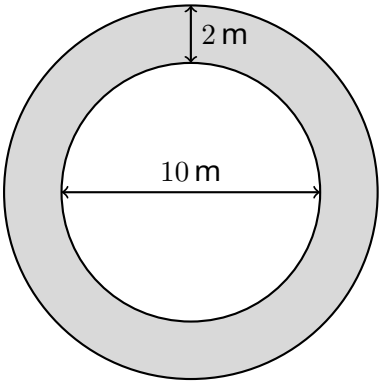


Diagram **NOT** accurately drawn

The diameter of the circular pond is 10 m.

Calculate the area of the path.
Give your answer as a multiple of π.

.....m^2

(Total for Question is 3 marks)

You Do 1

Katie designs a path in the corner of her garden. The path, shown shaded in the diagram, is the region between two quarter-circles with the same centre O.

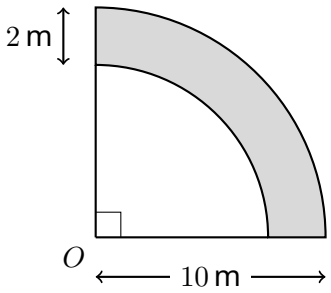


Diagram **NOT** accurately drawn

The radius of the outer quarter-circle is 10 m.
The path has a constant width of 2 m.

Calculate the area of the path.
Give your answer as a multiple of π.

.....m^2

(Total for Question is 3 marks)

You Do 2

Ryan is building a curved patio path shaped as the region between two concentric circular sectors of angle $120°$, as shown shaded below.

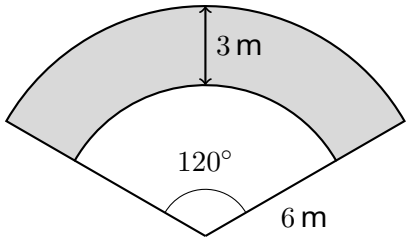


Diagram **NOT** accurately drawn

The inner sector has a radius of 6 m.
The path has a constant width of 3 m.

Calculate the area of the path.
Give your answer as a multiple of π.

.....m^2

(Total for Question is 3 marks)

You Do 3

Emma is designing a running track path around a rectangular grass field with semicircular ends, as shown in the diagram.

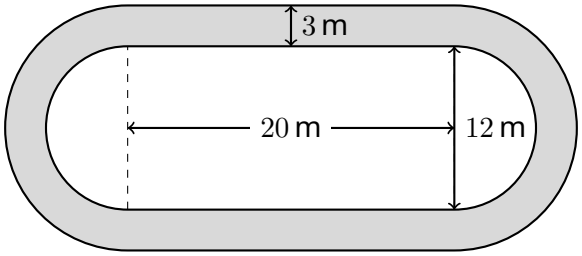


Diagram **NOT** accurately drawn

The inner straight sections are each 20 m long, and the inner semicircular ends each have a diameter of 12 m.
The path has a constant width of 3 m all the way around.

Calculate the total area of the path.
Give your answer in the form $a + b\pi$, where a and b are integers.

.....m^2

(Total for Question is 4 marks)

Challenge 1

Tom builds a path around three-quarters of a circular lawn. The path, shown shaded below, is the region between two concentric $270°$ sectors.

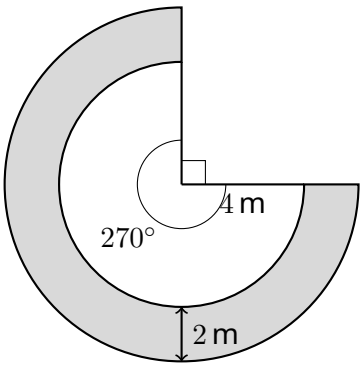


Diagram **NOT** accurately drawn

The inner radius of the path is 4 m, and the path has a constant width of 2 m.

Tom is going to cover the entire path with gravel.
One bag of gravel covers $5\,\text{m}^2$ and costs £12.50.
Bags of gravel can only be bought whole.

Calculate the total cost of the gravel Tom needs to buy.
You must show all your working clearly.

£ .....

(Total for Question is 5 marks)

Challenge 2

Catherine designs a semicircular path of constant width x m around a semicircular lawn of radius 6 m.

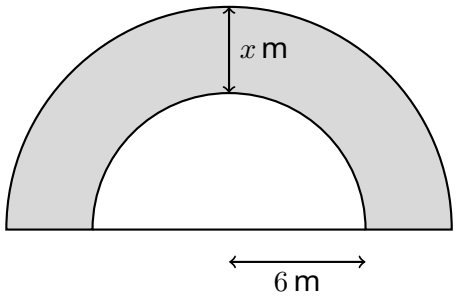


Diagram **NOT** accurately drawn

The area of the shaded path is 32π m^2.

Work out the value of x.
Show clear algebraic working.

$x =$ .....

(Total for Question is 4 marks)

Extreme Challenge

Mark is designing a curved garden path shaped as the region between two concentric circular sectors of angle $120°$, as shown shaded below.

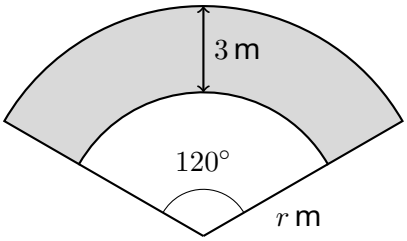


Diagram **NOT** accurately drawn

The inner unshaded sector is a lawn of radius r m.
The shaded path has a constant width of 3 m.

The area of the shaded path is equal to the area of the inner lawn.

Find the exact value of r.
Give your answer in the form $a + b\sqrt{c}$, where $a, b,$ and c are integers.
You must show all your algebraic working clearly.

$r =$ .....

(Total for Question is 5 marks)

Worked Answers

I Do (Sophie)
Outer diameter $= 12\,\text{m} \implies$ Outer radius $R = 6\,\text{m}$.
Inner radius $r = 6 - 2 = 4\,\text{m}$.
Area of outer semicircle $= \frac{1}{2} \times \pi \times 6^2 = 18\pi\,\text{m}^2$.
Area of inner semicircle $= \frac{1}{2} \times \pi \times 4^2 = 8\pi\,\text{m}^2$.
Area of path $= 18\pi - 8\pi =$ **10π m²**.

We Do (Ben)
Inner diameter $= 10\,\text{m} \implies$ Inner radius $r = 5\,\text{m}$.
Outer radius $R = 5 + 2 = 7\,\text{m}$.
Area of outer circle $= \pi \times 7^2 = 49\pi\,\text{m}^2$.
Area of inner circle $= \pi \times 5^2 = 25\pi\,\text{m}^2$.
Area of path $= 49\pi - 25\pi =$ **24π m²**.

You Do 1 (Katie)
Outer radius $R = 10\,\text{m}$.
Inner radius $r = 10 - 2 = 8\,\text{m}$.
Area of path $= \frac{1}{4}\pi R^2 - \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(10^2 - 8^2) = \frac{1}{4}\pi(100 - 64) = \frac{1}{4}\pi(36) =$ **9π m²**.

You Do 2 (Ryan)
Inner radius $r = 6\,\text{m}$.
Outer radius $R = 6 + 3 = 9\,\text{m}$.
Fraction of full circle $= \frac{120}{360} = \frac{1}{3}$.
Area of path $= \frac{1}{3}\pi(9^2 - 6^2) = \frac{1}{3}\pi(81 - 36) = \frac{1}{3}\pi(45) =$ **15π m²**.

You Do 3 (Emma)
1. Straight sections: two rectangles each measuring $20\,\text{m}$ by $3\,\text{m}$.
Area of straight sections $= 2 \times (20 \times 3) = 120\,\text{m}^2$.
2. Curved ends: two semicircular ends combine to form one full circular ring.
Inner radius $r = 12 \div 2 = 6\,\text{m}$.
Outer radius $R = 6 + 3 = 9\,\text{m}$.
Area of curved ends $= \pi(9^2 - 6^2) = \pi(81 - 36) = 45\pi\,\text{m}^2$.
3. Total area of path $=$ **$120 + 45\pi$ m²**.

Challenge 1 (Tom)
1. Find the area of the path:
Inner radius $r = 4$ m, Outer radius $R = 4 + 2 = 6$ m.
Fraction of full circle $= \frac{270}{360} = \frac{3}{4}$.
Area $= \frac{3}{4}\pi(6^2 - 4^2) = \frac{3}{4}\pi(36 - 16) = \frac{3}{4}\pi(20) = 15\pi$ m^2.
2. Convert to a decimal:
$15\pi = 47.1238\ldots$ m^2.
3. Find the number of bags needed:
$47.1238\cdots \div 5 = 9.4247\ldots$
Since bags can only be bought whole, Tom must buy 10 bags.
4. Total cost $= 10 \times$ £$12.50 =$ £**125**.

Challenge 2 (Catherine)
Inner radius $= 6$ m.
Outer radius $= (6 + x)$ m.
Set up the equation for the area of the semicircular path:
$\frac{1}{2}\pi(6 + x)^2 - \frac{1}{2}\pi(6)^2 = 32\pi$
Divide every term by π and multiply by 2:
$(6 + x)^2 - 36 = 64$
Add 36 to both sides:
$(6 + x)^2 = 100$
Take the square root of both sides (since $x > 0$, we only take the positive root):
$6 + x = 10 \implies$ **x** $=$ **4**.

Extreme Challenge (Mark)
Inner radius $= r$, Outer radius $= r + 3$.
Fraction of full circle $= \frac{120}{360} = \frac{1}{3}$.
Area of inner lawn $= \frac{1}{3}\pi r^2$.
Area of shaded path $= \frac{1}{3}\pi(r + 3)^2 - \frac{1}{3}\pi r^2$.
Set the area of the path equal to the area of the inner lawn:
$\frac{1}{3}\pi(r + 3)^2 - \frac{1}{3}\pi r^2 = \frac{1}{3}\pi r^2$
Divide every term by $\frac{1}{3}\pi$:
$(r + 3)^2 - r^2 = r^2$
Expand $(r + 3)^2$:
$r^2 + 6r + 9 - r^2 = r^2$
$6r + 9 = r^2$
Rearrange into a quadratic equal to zero:
$r^2 - 6r - 9 = 0$
Complete the square (or use the quadratic formula):
$(r - 3)^2 - 9 - 9 = 0$
$(r - 3)^2 = 18$
$r - 3 = \pm\sqrt{18} = \pm 3\sqrt{2}$
$r = 3 \pm 3\sqrt{2}$.
Since r is a length, $r > 0$ (and $3 - 3\sqrt{2} < 0$), so **r** $= \mathbf{3} + \mathbf{3}\sqrt{\mathbf{2}}$.