Bearings & Advanced Trigonometry

Practice Worksheet (Edexcel 4MA1 Style)

Exam Formulae:
Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$
Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

I Do

Ship A and Ship B both leave Port P at the same time.
Ship A sails 35 km on a bearing of 052°.
Ship B sails 58 km on a bearing of 118°.

Calculate the bearing of Ship B from Ship A.
Give your answer correct to the nearest degree.
Show all of your working clearly.

. . . . . °

(Total for Question is 4 marks)

We Do

Ship C and Ship D both leave Port P at the same time.
Ship C sails 42 km on a bearing of 065°.
Ship D sails 70 km on a bearing of 140°.

Calculate the bearing of Ship D from Ship C.
Give your answer correct to the nearest degree.
Show all of your working clearly.

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(Total for Question is 4 marks)

You Do 1

Ship E and Ship F both leave Port P at the same time.
Ship E sails 25 km on a bearing of 040°.
Ship F sails 45 km on a bearing of 105°.

Calculate the bearing of Ship F from Ship E.
Give your answer correct to the nearest degree.
Show all of your working clearly.

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(Total for Question is 4 marks)

You Do 2

Ship G and Ship H both leave Port P at the same time.
Ship G sails 60 km on a bearing of 120°.
Ship H sails 95 km on a bearing of 210°.

Calculate the bearing of Ship H from Ship G.
Give your answer correct to the nearest degree.
Show all of your working clearly.

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(Total for Question is 4 marks)

You Do 3

Ship J and Ship K both leave Port P at the same time.
Ship J sails 40 km on a bearing of 300°.
Ship K sails 75 km on a bearing of 045°.

Calculate the bearing of Ship K from Ship J.
Give your answer as a three-digit bearing correct to the nearest degree.
Show all of your working clearly.

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(Total for Question is 4 marks)

Challenge 1

Two ships leave a port at 12:00.
Ship X sails on a bearing of 070° at a constant speed of 15 km/h.
Ship Y sails on a bearing of 150° at a constant speed of 22 km/h.

After exactly 3 hours, Ship X stops and sends a distress signal.
Ship Y immediately turns and sails directly to Ship X at a constant speed of 25 km/h.

Calculate the time it takes for Ship Y to reach Ship X.
Give your answer in hours and minutes, to the nearest minute.
Show all your working clearly.

..... hours ..... minutes

(Total for Question is 5 marks)

Challenge 2

Ship M leaves Port P and sails 40 km on a bearing of $x°$, where $0 < x < 90$.
Ship N leaves Port P and sails 65 km on a bearing of 145°.

The direct distance between Ship M and Ship N is exactly 80 km.

Find the value of x.
Give your answer correct to the nearest whole number.
You must show clear algebraic working.

$x =$ .....

(Total for Question is 5 marks)

Extreme Challenge

Three ports A, B, and C are located such that:
Port B is 50 km from Port A on a bearing of 040°.
Port C is 80 km from Port A on a bearing of 115°.

Calculate the bearing of Port B from Port C.
Give your answer correct to the nearest degree.
You must show all your working clearly, including a sketch of the positions.

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(Total for Question is 6 marks)

Worked Answers

I Do

1. Find the angle at Port P: $\angle P = 118° - 52° = 66°$.
2. Use the Cosine Rule to find the distance AB:
$AB^2 = 35^2 + 58^2 - 2(35)(58)\cos(66°)$
$AB^2 = 1225 + 3364 - 4060(0.4067...) = 4589 - 1651.35... = 2937.64...$
$AB = \sqrt{2937.64...} = 54.199...\,\text{km}$.
3. Use the Sine Rule to find the internal angle at A (angle PAB):
$\frac{\sin A}{58} = \frac{\sin(66°)}{54.199...} \implies \sin A = \frac{58 \times \sin(66°)}{54.199...} = 0.9776...$
$\angle PAB = \sin^{-1}(0.9776...) = 77.85...°$.
4. Find the bearing:
The bearing of P from A is $180° + 052° = 232°$.
Ship B is to the left (counter-clockwise) when looking from A to P.
Bearing of B from $A = 232° - 77.85° = 154.15°$.
Correct to nearest degree: **154**°.

We Do

1. $\angle P = 140° - 65° = 75°$.
2. Distance CD: $CD^2 = 42^2 + 70^2 - 2(42)(70)\cos(75°)$
$CD^2 = 1764 + 4900 - 5880(0.2588...) = 6664 - 1521.85... = 5142.14...$
$CD = 71.708...\,\text{km}$.
3. Internal angle at C (angle PCD):
$\frac{\sin C}{70} = \frac{\sin(75°)}{71.708...} \implies \sin C = 0.9429... \implies \angle PCD = 70.54°$.
4. The bearing of P from C is $180° + 065° = 245°$.
Bearing of D from $C = 245° - 70.54° = 174.46°$.
Correct to nearest degree: **174**°.

You Do 1

1. $\angle P = 105° - 40° = 65°$.
2. Distance EF: $EF^2 = 25^2 + 45^2 - 2(25)(45)\cos(65°)$
$EF^2 = 625 + 2025 - 2250(0.4226...) = 2650 - 950.89... = 1699.10...$
$EF = 41.220...\,\text{km}$.
3. Internal angle at E (angle PEF):
$\frac{\sin E}{45} = \frac{\sin(65°)}{41.220...} \implies \sin E = 0.9894... \implies \angle PEF = 81.68°$.
4. The bearing of P from E is $180° + 040° = 220°$.
Bearing of F from $E = 220° - 81.68° = 138.32°$.
Correct to nearest degree: **138**°.

You Do 2

1. $\angle P = 210° - 120° = 90°$ (This is a right-angled triangle!).
2. Distance GH: $GH^2 = 60^2 + 95^2 = 3600 + 9025 = 12625 \implies GH = 112.36...\,\text{km}$.
3. Internal angle at G (angle PGH):
Since it is a right-angled triangle, we can use SOH CAH TOA.
$\tan G = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{95}{60} \implies \angle PGH = \tan^{-1}\left(\frac{95}{60}\right) = 57.72°$.
4. The bearing of P from G is $180° + 120° = 300°$.
Bearing of H from $G = 300° - 57.72° = 242.28°$.
Correct to nearest degree: **242**°.

You Do 3

1. The angle between them at P crosses North.
$\angle P = (360° - 300°) + 045° = 60° + 45° = 105°$.
2. Distance JK: $JK^2 = 40^2 + 75^2 - 2(40)(75)\cos(105°)$
$JK^2 = 1600 + 5625 - 6000(-0.2588...) = 7225 + 1552.91... = 8777.91...$
$JK = 93.690...\,\text{km}$.
3. Internal angle at J (angle PJK):
$\frac{\sin J}{75} = \frac{\sin(105°)}{93.690...} \implies \sin J = 0.7732... \implies \angle PJK = 50.64°$.
4. The bearing of P from J is $300° - 180° = 120°$.
Bearing of K from $J = 120° - 50.64° = 69.36°$.
Correct to nearest degree and formatted as 3 digits: **069**°.

Challenge 1

1. Distance traveled in 3 hours:
$PX = 3 \times 15 = 45\,\text{km}$.
$PY = 3 \times 22 = 66\,\text{km}$.
2. Angle at port P: $\angle P = 150° - 070° = 80°$.
3. Calculate distance XY using the Cosine Rule:
$XY^2 = 45^2 + 66^2 - 2(45)(66)\cos(80°)$
$XY^2 = 2025 + 4356 - 5940(0.1736...) = 6381 - 1031.47... = 5349.52...$
$XY = \sqrt{5349.52...} = 73.140...\,\text{km}$.
4. Calculate time taken by Ship Y to reach Ship X:
Time $= \frac{\text{Distance}}{\text{Speed}} = \frac{73.140...}{25} = 2.9256...$ hours.
Convert decimal hours to minutes: $0.9256... \times 60 = 55.53...$ minutes.
Total time is **2 hours and 56 minutes**.

Challenge 2

1. Set up the Cosine Rule to find the angle at P (angle MPN):
$MN^2 = PM^2 + PN^2 - 2(PM)(PN)\cos P$
$80^2 = 40^2 + 65^2 - 2(40)(65)\cos P$
$6400 = 1600 + 4225 - 5200\cos P$
$6400 = 5825 - 5200\cos P$
$575 = -5200\cos P \implies \cos P = -\frac{575}{5200} = -0.11057...$
2. Find the angle P:
$P = \cos^{-1}(-0.11057...) = 96.35°$.
3. Calculate bearing x:
Ship N is at a bearing of 145°. Ship M is separated from N by 96.35°.
Since $0 < x < 90$, Ship M must be counter-clockwise from Ship N.
$x = 145° - 96.35° = 48.65°$.
Nearest whole number: $\mathbf{x = 49}$.

Extreme Challenge

1. Calculate angle at A: $\angle BAC = 115° - 040° = 75°$.
2. Use Cosine Rule to find distance BC:
$BC^2 = 50^2 + 80^2 - 2(50)(80)\cos(75°)$
$BC^2 = 2500 + 6400 - 8000(0.2588...) = 8900 - 2070.55... = 6829.44...$
$BC = 82.640...$ km.
3. Use Sine Rule to find the internal angle at C (angle BCA):
$\frac{\sin C}{50} = \frac{\sin(75°)}{82.640...} \implies \sin C = \frac{50 \times \sin(75°)}{82.640...} = 0.5844...$
$\angle BCA = \sin^{-1}(0.5844...) = 35.76°$.
4. Calculate the bearing:
The bearing of A from C is $115° + 180° = 295°$.
When looking from C towards A (at 295°), Port B sits further clockwise (it is North of A, while C is South-East of A).
Bearing of B from $C = 295° + 35.76° = 330.76°$.
Correct to nearest degree: $\mathbf{331}°$.