Completing the Square & Quadratic Graphs

Practice Worksheet (Problem Solving)

I Do

A rectangle has length $(10 - x)$ cm and width $(3x + 6)$ cm.

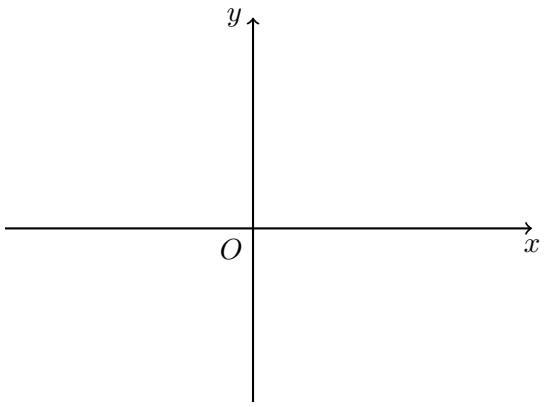
(a) Show that the area of the rectangle, A cm^2, is given by $A = 60 + 24x - 3x^2$.

(2 marks)

(b) By completing the square, find the maximum possible area of this rectangle, and the exact value of x for which this maximum area occurs.

Maximum area = ...cm^2 at x = ... *(4 marks)*

(c) Hence, sketch the graph of $y = -3x^2 + 24x + 60$, clearly showing the coordinates of the turning point and the exact coordinates of any intercepts with the coordinate axes.



(3 marks)

(Total for Question is 9 marks)

We Do

A rectangle has length $(8 - x)$ cm and width $(2x + 4)$ cm.

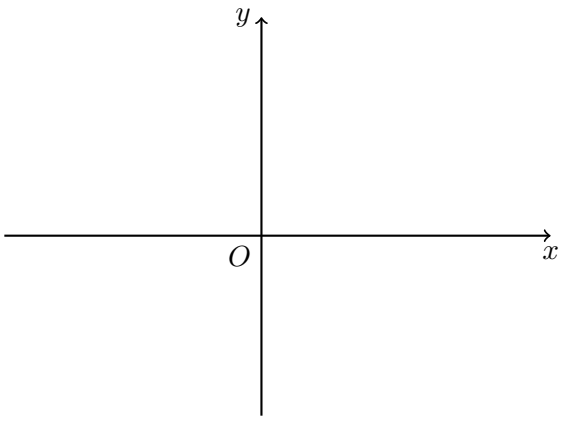
(a) Show that the area of the rectangle, A cm^2, is given by $A = 32 + 12x - 2x^2$.

(2 marks)

(b) By completing the square, find the maximum possible area of this rectangle, and the exact value of x for which this maximum area occurs.

Maximum area = ...cm^2 at x = ... *(4 marks)*

(c) Hence, sketch the graph of $y = -2x^2 + 12x + 32$, clearly showing the coordinates of the turning point and the exact coordinates of any intercepts with the coordinate axes.



(3 marks)

(Total for Question is 9 marks)

You Do 1

A rectangle has length $(9 - x)$ cm and width $(2x + 2)$ cm.

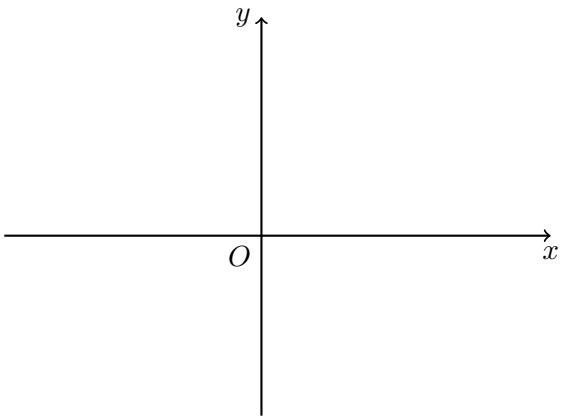
(a) Show that the area of the rectangle, A cm^2, is given by $A = 18 + 16x - 2x^2$.

(2 marks)

(b) By completing the square, find the maximum possible area of this rectangle, and the exact value of x for which this maximum area occurs.

Maximum area = ...cm^2 at x = ... *(4 marks)*

(c) Hence, sketch the graph of $y = -2x^2 + 16x + 18$, clearly showing the coordinates of the turning point and the exact coordinates of any intercepts with the coordinate axes.



(3 marks)

(Total for Question is 9 marks)

You Do 2

A rectangle has length $(7 - x)$ cm and width $(3x + 1)$ cm.

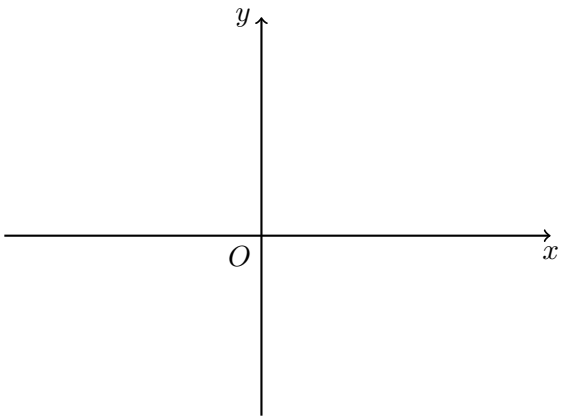
(a) Show that the area of the rectangle, A cm^2, is given by $A = 7 + 20x - 3x^2$.

(2 marks)

(b) By completing the square, find the maximum possible area of this rectangle, and the exact value of x for which this maximum area occurs.

Maximum area = ...cm^2 at x = ... *(4 marks)*

(c) Hence, sketch the graph of $y = -3x^2 + 20x + 7$, clearly showing the coordinates of the turning point and the exact coordinates of any intercepts with the coordinate axes.



(3 marks)

(Total for Question is 9 marks)

You Do 3

A rectangle has length $(5 - x)$ cm and width $(4x + 3)$ cm.

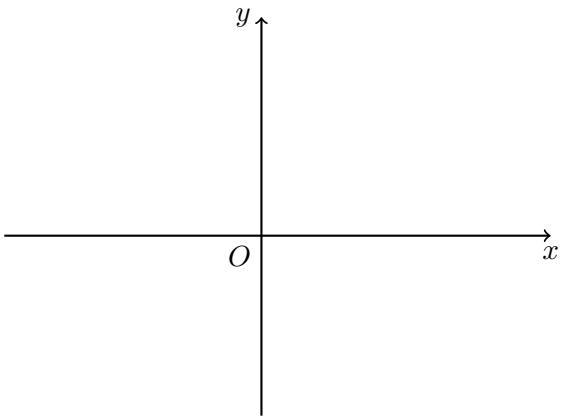
(a) Show that the area of the rectangle, A cm^2, is given by $A = 15 + 17x - 4x^2$.

(2 marks)

(b) By completing the square, find the maximum possible area of this rectangle, and the exact value of x for which this maximum area occurs.

Maximum area = ...cm^2 at $x =$ ..... *(4 marks)*

(c) Hence, sketch the graph of $y = -4x^2 + 17x + 15$, clearly showing the coordinates of the turning point and the exact coordinates of any intercepts with the coordinate axes.



(3 marks)

(Total for Question is 9 marks)

Worked Answers

I Do

(a) $A = (10 - x)(3x + 6) = 30x + 60 - 3x^2 - 6x = 60 + 24x - 3x^2$.
(b) Factor out -3: $A = -3(x^2 - 8x) + 60$.
$A = -3[(x - 4)^2 - 16] + 60 = -3(x - 4)^2 + 108$.
Maximum area $= 108$ cm^2, occurring at $x = 4$.
(c) Downward parabola with turning point $(4, 108)$ and y-intercept $(0, 60)$.
Set $(10 - x)(3x + 6) = 0$: the x-intercepts are $(-2, 0)$ and $(10, 0)$.

(a) $A = (8 - x)(2x + 4) = 16x + 32 - 2x^2 - 4x = 32 + 12x - 2x^2$.
(b) Factor out -2: $A = -2(x^2 - 6x) + 32$.
$A = -2[(x - 3)^2 - 9] + 32 = -2(x - 3)^2 + 50$.
Maximum area $= 50$ cm^2, occurring at $x = 3$.
(c) Downward parabola with turning point $(3, 50)$ and y-intercept $(0, 32)$.
Set $(8 - x)(2x + 4) = 0$: the x-intercepts are $(-2, 0)$ and $(8, 0)$.

You Do 1

(a) $A = (9 - x)(2x + 2) = 18x + 18 - 2x^2 - 2x = 18 + 16x - 2x^2$.
(b) $A = -2(x^2 - 8x) + 18 = -2\left[(x - 4)^2 - 16\right] + 18 = -2(x - 4)^2 + 32 + 18 = -2(x - 4)^2 + 50$.
Maximum area $=$ **50 cm^2**, occurring at **$x = 4$**.
(c) Inverted ($\cap$-shaped) parabola with:
- Turning point: (**4**, **50**) - y-intercept: (**0**, **18**) - x-intercepts: ($-$**1**, **0**) and (**9**, **0**).

You Do 2

(a) $A = (7 - x)(3x + 1) = 21x + 7 - 3x^2 - x = 7 + 20x - 3x^2$.

(b) $A = -3\left(x^2 - \frac{20}{3}x\right) + 7 = -3\left[\left(x - \frac{10}{3}\right)^2 - \frac{100}{9}\right] + 7$

$A = -3\left(x - \frac{10}{3}\right)^2 + \frac{100}{3} + \frac{21}{3} = -3\left(x - \frac{10}{3}\right)^2 + \frac{121}{3}$.

Maximum area $= \mathbf{\frac{121}{3}}$ **cm²** (or $40\frac{1}{3}$ cm^2), occurring at **x** $= \mathbf{\frac{10}{3}}$ (or $3\frac{1}{3}$).

(c) Inverted (∩-shaped) parabola with:

- Turning point: $\left(\mathbf{\frac{10}{3}}, \mathbf{\frac{121}{3}}\right)$
- y-intercept: $(\mathbf{0}, \mathbf{7})$
- x-intercepts (set $(7 - x)(3x + 1) = 0$): $\left(-\mathbf{\frac{1}{3}}, \mathbf{0}\right)$ and $(\mathbf{7}, \mathbf{0})$.

You Do 3

(a) $A = (5 - x)(4x + 3) = 20x + 15 - 4x^2 - 3x = 15 + 17x - 4x^2$.

(b) $A = -4\left(x^2 - \frac{17}{4}x\right) + 15 = -4\left[\left(x - \frac{17}{8}\right)^2 - \frac{289}{64}\right] + 15$

$A = -4\left(x - \frac{17}{8}\right)^2 + \frac{289}{16} + \frac{240}{16} = -4\left(x - \frac{17}{8}\right)^2 + \frac{529}{16}$.

Maximum area $= \mathbf{\frac{529}{16}}$ **cm²** (or 33.0625 cm^2), occurring at **x** $= \mathbf{\frac{17}{8}}$ (or 2.125).

(c) Inverted (∩-shaped) parabola with:

- Turning point: $\left(\mathbf{\frac{17}{8}}, \mathbf{\frac{529}{16}}\right)$
- y-intercept: $(\mathbf{0}, \mathbf{15})$
- x-intercepts (set $(5 - x)(4x + 3) = 0$): $\left(-\mathbf{\frac{3}{4}}, \mathbf{0}\right)$ and $(\mathbf{5}, \mathbf{0})$.