Kinematics (Calculus)

Practice Worksheet (Edexcel 4MA1 Style)

I Do

A particle P moves along a straight line.
The displacement, s metres, of P from a fixed point O at time t seconds is given by:

$$s = t^3 - 7.5t^2 + 12t + 4$$

(a) Find an expression for the velocity, v m/s, of P at time t.

. . . . . *(2 marks)*

(b) Find the time at which the acceleration of P is 0 m/s^2.

$t =$ . . . . . *(3 marks)*

(Total for Question is 5 marks)

We Do

A particle Q moves along a straight line.
The displacement, s metres, of Q from a fixed point O at time t seconds is given by:

$$s = t^3 - 6t^2 + 9t - 2$$

(a) Find an expression for the velocity, v m/s, of Q at time t.

..... *(2 marks)*

(b) Find the time at which the acceleration of Q is 0 m/s^2.

$t =$ ..... *(3 marks)*

(Total for Question is 5 marks)

You Do 1

A particle A moves along a straight line.
The displacement, s metres, of A from a fixed point O at time t seconds is given by:

$$s = t^3 - 4.5t^2 + 6t$$

(a) Find an expression for the velocity, v m/s, of A at time t.

..... *(2 marks)*

(b) Find the time at which the acceleration of A is 0 m/s^2.

$t =$ ..... *(3 marks)*

(Total for Question is 5 marks)

You Do 2

A particle B moves along a straight line.
The displacement, s metres, of B from a fixed point O at time t seconds is given by:

$$s = 2t^3 - 12t^2 + 15t - 7$$

(a) Find an expression for the velocity, v m/s, of B at time t.

..... *(2 marks)*

(b) Find the time at which the acceleration of B is 0 m/s^2.

$t =$ ..... *(3 marks)*

(Total for Question is 5 marks)

You Do 3

A particle C moves along a straight line.
The displacement, s metres, of C from a fixed point O at time t seconds is given by:

$$s = \frac{1}{3}t^3 - 2.5t^2 + 4t + 1$$

(a) Find an expression for the velocity, v m/s, of C at time t.

..... *(2 marks)*

(b) Find the time at which the acceleration of C is 0 m/s^2.

$t =$ ..... *(3 marks)*

(Total for Question is 5 marks)

Challenge 1

A particle P moves along a straight line such that its displacement, s metres, from a fixed point O at time t seconds is given by:

$$s = t^3 - 6t^2 + 9t$$

The particle is instantaneously at rest at two different times.

Calculate the exact distance between the two points where the particle is instantaneously at rest.
You must show all your algebraic working clearly.

.....m

(Total for Question is 5 marks)

Challenge 2

A particle Q moves along a straight line such that its displacement, s metres, from a fixed point O at time t seconds is given by:

$$s = \frac{2}{3}t^3 - 7t^2 + 20t + 5$$

Find the range of values for t during which the particle is moving with a negative velocity. Show clear algebraic working to justify your answer.

.....

(Total for Question is 5 marks)

Extreme Challenge

Two particles, A and B, move along the same straight line.
The displacement, s_A metres, of particle A from a fixed point O at time t seconds is given by:

$$s_A = t^3 - 6t^2 + 9t$$

The displacement, s_B metres, of particle B from the same fixed point O at time t seconds is given by:

$$s_B = 3t^2 - 6t$$

There are two times, $t > 0$, when the two particles have exactly the same velocity.

Find the distance between the two particles at the later of these two times.
You must show all your algebraic working clearly.

..... m

(Total for Question is 6 marks)

Worked Answers

I Do

(a) To find velocity, differentiate displacement s with respect to t:
$v = \frac{ds}{dt} = 3 \times t^2 - 2 \times 7.5t^1 + 12$
$\mathbf{v = 3t^2 - 15t + 12}$
(b) To find acceleration, differentiate velocity v with respect to t:
$a = \frac{dv}{dt} = 2 \times 3t - 15$
$a = 6t - 15$
Set the acceleration to 0:
$6t - 15 = 0 \implies 6t = 15 \implies t = \frac{15}{6} = \mathbf{2.5}$ seconds.

We Do

(a) $v = \frac{ds}{dt} = \mathbf{3t^2 - 12t + 9}$
(b) $a = \frac{dv}{dt} = 6t - 12$
Set $a = 0$:
$6t - 12 = 0 \implies 6t = 12 \implies \mathbf{t = 2}$.

You Do 1

(a) $v = \frac{ds}{dt} = \mathbf{3t^2 - 9t + 6}$
(b) $a = \frac{dv}{dt} = 6t - 9$
Set $a = 0$:
$6t - 9 = 0 \implies 6t = 9 \implies \mathbf{t = 1.5}$.

You Do 2

(a) $v = \frac{ds}{dt} = 3 \times 2t^2 - 2 \times 12t + 15$
$\mathbf{v = 6t^2 - 24t + 15}$
(b) $a = \frac{dv}{dt} = 12t - 24$
Set $a = 0$:
$12t - 24 = 0 \implies 12t = 24 \implies \mathbf{t = 2}$.

You Do 3

(a) $v = \frac{ds}{dt} = 3 \times \frac{1}{3}t^2 - 2 \times 2.5t + 4$
$\mathbf{v = t^2 - 5t + 4}$
(b) $a = \frac{dv}{dt} = 2t - 5$
Set $a = 0$:
$2t - 5 = 0 \implies 2t = 5 \implies \mathbf{t = 2.5}$.

Challenge 1

1. The particle is instantaneously at rest when its velocity is zero ($v = 0$).
$v = \frac{ds}{dt} = 3t^2 - 12t + 9$
$3t^2 - 12t + 9 = 0$
2. Divide by 3 and factorise to solve for t:
$t^2 - 4t + 3 = 0$
$(t - 1)(t - 3) = 0 \implies t = 1$ and $t = 3$.
3. Find the displacement (s) at these two times:
At $t = 1$: $s = (1)^3 - 6(1)^2 + 9(1) = 1 - 6 + 9 = 4$ m.
At $t = 3$: $s = (3)^3 - 6(3)^2 + 9(3) = 27 - 54 + 27 = 0$ m.
4. Calculate the distance between these two points:
Distance $= 4 - 0 = \mathbf{4\ m}$.

Challenge 2

1. Find the expression for velocity:
$v = \frac{ds}{dt} = 3 \times \frac{2}{3}t^2 - 2 \times 7t + 20 = 2t^2 - 14t + 20.$
2. We need to find when $v < 0$:
$2t^2 - 14t + 20 < 0$
3. Divide by 2:
$t^2 - 7t + 10 < 0$
4. Factorise to find the critical values:
$(t-2)(t-5) = 0 \implies t = 2$ and $t = 5$.
5. Since it's a positive quadratic (U-shape), the expression is negative between the roots.
$\mathbf{2 < t < 5}$.

Extreme Challenge

1. Find expressions for the velocity of both particles:
$v_A = \frac{ds_A}{dt} = 3t^2 - 12t + 9$
$v_B = \frac{ds_B}{dt} = 6t - 6$
2. Set their velocities equal to each other to find the times:
$3t^2 - 12t + 9 = 6t - 6$
$3t^2 - 18t + 15 = 0$
Divide by 3: $t^2 - 6t + 5 = 0$
Factorise: $(t-1)(t-5) = 0 \implies t = 1$ and $t = 5$.
The later time is $t = 5$.
3. Find the displacement of both particles at $t = 5$:
$s_A = (5)^3 - 6(5)^2 + 9(5) = 125 - 150 + 45 = 20$ m.
$s_B = 3(5)^2 - 6(5) = 3(25) - 30 = 75 - 30 = 45$ m.
4. Calculate the distance between them:
Distance $= 45 - 20 = \mathbf{25\ m}$.