Upper & Lower Bounds

Practice Worksheet (Problem Solving)

I Do

A train travels 230 km, correct to the nearest 10 km.
The journey takes 4.5 hours, correct to 1 decimal place.
Calculate the upper bound for the average speed, in km/h, to 3 significant figures.
Show your working clearly.

.....km/h

(Total for Question is 4 marks)

We Do

A car travels 180 km, correct to the nearest 10 km.
The journey takes 3.2 hours, correct to 1 decimal place.
Calculate the upper bound for the average speed, in km/h, to 3 significant figures.
Show your working clearly.

.....km/h

(Total for Question is 3 marks)

You Do 1

A coach travels 340 km, correct to the nearest 10 km.
The journey takes 5.5 hours, correct to 1 decimal place.
Calculate the **lower bound** for the average speed, in km/h, to 3 significant figures.
Show your working clearly.

.....km/h

(Total for Question is 3 marks)

You Do 2

A solid metal block has a mass of 460 g, correct to the nearest 10 g.
The volume of the block is 64 cm^3, correct to the nearest cm^3.

Calculate the upper bound for the density of the metal, in g/cm^3, to 3 significant figures.
Show your working clearly.

.....g/cm^3

(Total for Question is 3 marks)

You Do 3

$$y = \frac{a - b}{c}$$

$a = 14.6$, correct to 1 decimal place.
$b = 5.2$, correct to 1 decimal place.
$c = 2.4$, correct to 2 significant figures.

Calculate the upper bound for the value of y.
Give your answer correct to 3 significant figures.
Show your working clearly.

.....

(Total for Question is 3 marks)

Challenge 1

$$m = \frac{\sqrt{s}}{t}$$

$s = 5.28$, correct to 3 significant figures.
$t = 1.42$, correct to 3 significant figures.

By considering bounds, work out the value of m to a suitable degree of accuracy.
Give a reason for your answer.
You must show all of your working clearly.

$m =$ .....

(Total for Question is 4 marks)

Challenge 2

A trapezium has an area of $65\,\text{cm}^2$, correct to the nearest $5\,\text{cm}^2$.
Its two parallel sides have lengths $8.4\,\text{cm}$ and $11.6\,\text{cm}$, both correct to 1 decimal place.

Calculate the upper bound for the perpendicular height of the trapezium.
Give your answer correct to 3 significant figures.
Show all of your working clearly.

.....cm

(Total for Question is 4 marks)

Extreme Challenge

An empty water tank is in the shape of a cuboid with internal dimensions:

- length $= 120$ cm
- width $= 80$ cm
- height $= 50$ cm

Each dimension is measured correct to the nearest 10 cm.

Water is pumped into the tank at a constant rate of 15 litres per minute, correct to the nearest litre per minute. *(1 litre $= 1000\,cm^3$)*

(a) Calculate the upper bound for the time, in minutes, taken to fill the tank completely.
Give your answer correct to 3 significant figures.

.....minutes *(4 marks)*

(b) A claim is made that it is **impossible** for the tank to take more than 40 minutes to fill.
Is this claim correct? Give a reason for your answer.

(1 mark)

(Total for Question is 5 marks)

Worked Answers

I Do

Bounds for distance (230 km to nearest 10 km): $LB = 225$ km, $\quad UB = 235$ km.
Bounds for time (4.5 hours to 1 d.p.): $LB = 4.45$ hours, $\quad UB = 4.55$ hours.
To maximise Speed $= \dfrac{\text{Distance}}{\text{Time}}$, divide the upper bound of distance by the lower bound of time:
Upper bound $= \dfrac{235}{4.45} = 52.8089\cdots \approx$ **52.8 km/h** *(3 s.f.)*.

We Do

$UB(\text{Distance}) = 185$ km, $\quad LB(\text{Time}) = 3.15$ hours.
Upper bound $= \dfrac{185}{3.15} = 58.7301\cdots \approx$ **58.7 km/h** *(3 s.f.)*.

You Do 1

To minimise speed, divide the lower bound of distance by the upper bound of time:
$LB(\text{Distance}) = 335$ km, $\quad UB(\text{Time}) = 5.55$ hours.
Lower bound $= \dfrac{335}{5.55} = 60.3603\cdots \approx$ **60.4 km/h** *(3 s.f.)*.

You Do 2

Density $= \dfrac{\text{Mass}}{\text{Volume}}$.
$UB(\text{Mass}) = 465$ g, $\quad LB(\text{Volume}) = 63.5$ cm^3.
Upper bound $= \dfrac{465}{63.5} = 7.3228\cdots \approx$ **7.32 g/cm^3** *(3 s.f.)*.

You Do 3

To maximise $y = \dfrac{a-b}{c}$, maximise the numerator ($UB(a) - LB(b)$) and minimise the denominator ($LB(c)$):
$UB(a) = 14.65$, $\quad LB(b) = 5.15$, $\quad LB(c) = 2.35$.
$UB(y) = \dfrac{14.65 - 5.15}{2.35} = \dfrac{9.5}{2.35} = 4.0425\cdots \approx$ **4.04** *(3 s.f.)*.

Challenge 1

1. Find the bounds for s and t:
$LB(s) = 5.275, \quad UB(s) = 5.285.$
$LB(t) = 1.415, \quad UB(t) = 1.425.$
2. Calculate the upper bound of m:
$$UB(m) = \frac{\sqrt{5.285}}{1.415} = \frac{2.29891\ldots}{1.415} = 1.62467\ldots$$
3. Calculate the lower bound of m:
$$LB(m) = \frac{\sqrt{5.275}}{1.425} = \frac{2.29673\ldots}{1.425} = 1.61174\ldots$$
4. Compare both bounds:
Both the upper bound ($1.6246\ldots$) and the lower bound ($1.6117\ldots$) round to **1.6** correct to 2 significant figures (they differ at 3 significant figures: 1.62 vs 1.61).
Answer: m $= 1.6$ *(correct to 2 significant figures, as both the upper and lower bounds round to 1.6 to 2 s.f.).*

Challenge 2

1. Rearrange the trapezium area formula $A = \frac{1}{2}(a+b)h$ to make h the subject:
$h = \frac{2A}{a+b}$.
2. To maximise h, use the upper bound of A and the lower bounds of a and b:
$UB(A) = 65 + 2.5 = 67.5\,\text{cm}^2$.
$LB(a) = 8.35\,\text{cm}, \quad LB(b) = 11.55\,\text{cm}.$
3. Substitute into the formula:
$UB(h) = \frac{2 \times 67.5}{8.35 + 11.55} = \frac{135}{19.9} = 6.7839\cdots \approx$ **6.78 cm** *(3 s.f.).*

Extreme Challenge

(a) 1. Find the upper bound of each internal dimension (nearest $10\,\text{cm} \implies \pm 5\,\text{cm}$):
$UB(\text{length}) = 125\,\text{cm}, \quad UB(\text{width}) = 85\,\text{cm}, \quad UB(\text{height}) = 55\,\text{cm}.$
2. Calculate the upper bound for the volume of the tank:
$UB(\text{Volume}) = 125 \times 85 \times 55 = 584\,375\,\text{cm}^3$.
Convert to litres: $584\,375 \div 1000 = 584.375$ litres.
3. Find the lower bound of the filling rate (nearest litre/min $\implies \pm 0.5$):
$LB(\text{Rate}) = 14.5$ litres per minute.
4. Calculate the upper bound for the time taken:
$$UB(\text{Time}) = \frac{584.375}{14.5} = 40.3017\cdots \approx \textbf{40.3 minutes} \textit{ (3 s.f.).}$$

(b) No, the claim is not correct. Because the upper bound for the filling time is 40.3 minutes (which is greater than 40 minutes), it is possible for the tank to take more than 40 minutes to fill.