# Surds: Rationalising the Denominator

*Practice Worksheet (Problem Solving)*

## I Do

**(a)** Show that the first expression below can be written in the stated form, where $a$ and $b$ are integers. Show your working clearly.

$$\frac{14}{3-\sqrt{5}} = \frac{a+b\sqrt{5}}{2}$$

..... *(3 marks)*

**(b)** Hence write the second expression below in the same form.

$$\frac{14}{3+\sqrt{5}}$$

..... *(1 mark)*

**(Total for Question is 4 marks)**

## We Do

**(a)** Show that the first expression below can be written in the stated form, where $a$ and $b$ are integers. Show your working clearly.

$$\frac{15}{3-\sqrt{3}} = \frac{a+b\sqrt{3}}{2}$$

..... *(3 marks)*

**(b)** Hence write the second expression below in the same form.

$$\frac{15}{3+\sqrt{3}}$$

..... *(1 mark)*

**(Total for Question is 4 marks)**

## You Do 1

**(a)** Show that the first expression below can be written in the stated form, where $a$ and $b$ are integers. Show your working clearly.

$$\frac{10}{4-\sqrt{10}} = \frac{a+b\sqrt{10}}{3}$$

..... *(3 marks)*

**(b)** Hence write the second expression below in the same form.

$$\frac{10}{4+\sqrt{10}}$$

..... *(1 mark)*

**(Total for Question is 4 marks)**

## You Do 2

**(a)** Show that the first expression below can be written in the stated form, where $a$ and $b$ are integers. Show your working clearly.

$$\frac{12}{5-\sqrt{17}} = \frac{a+b\sqrt{17}}{2}$$

..... *(3 marks)*

**(b)** Hence write the second expression below in the same form.

$$\frac{12}{5+\sqrt{17}}$$

..... *(1 mark)*

**(Total for Question is 4 marks)**

## You Do 3

**(a)** Rationalise the denominator of

$$\frac{3+\sqrt{2}}{4-\sqrt{2}}$$

Give your answer in the form $\frac{a+\sqrt{2}}{b}$, where $a$ and $b$ are integers.

..... *(3 marks)*

**(b)** Write

$$\frac{10}{\sqrt{5}}+\frac{6}{2-\sqrt{5}}$$

in the form $p+q\sqrt{5}$, where $p$ and $q$ are integers.

..... *(3 marks)*

**(Total for Question is 6 marks)**

## Challenge 1

Given that

$$\frac{6+\sqrt{12}}{3-\sqrt{3}}=a+b\sqrt{3}$$

where $a$ and $b$ are integers, find the value of $a$ and the value of $b$.
You must show all of your working clearly.

$a =$ ..... $b =$ .....

**(Total for Question is 4 marks)**

## Challenge 2

A rectangle has an area of $(11 + 5\sqrt{3})$ cm$^2$ and a width of $(2 + \sqrt{3})$ cm.

**(a)** Find the length of the rectangle in the form $(a + b\sqrt{3})$ cm, where $a$ and $b$ are integers.
Show all of your working clearly.

.....cm *(3 marks)*

**(b)** Hence, work out the exact perimeter of the rectangle.

.....cm *(2 marks)*

**(Total for Question is 5 marks)**

## Extreme Challenge

**(a)** Show that for any positive integer $n$,

$$\frac{1}{\sqrt{n} + \sqrt{n+1}} = \sqrt{n+1} - \sqrt{n}$$

*(2 marks)*

**(b)** Hence, find the exact value of

$$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots + \frac{1}{\sqrt{35}+\sqrt{36}}$$

You must show all of your working clearly.

..... *(3 marks)*

**(Total for Question is 5 marks)**

# Worked Answers

**I Do**

**(a)** Multiply numerator and denominator by the conjugate $(3+\sqrt{5})$:

$$\frac{14(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})} = \frac{14(3+\sqrt{5})}{9-5} = \frac{14(3+\sqrt{5})}{4} = \frac{7(3+\sqrt{5})}{2} = \mathbf{\frac{21+7\sqrt{5}}{2}}$$ ($a=21,\ b=7$).

**(b)** Replacing $-\sqrt{5}$ with $+\sqrt{5}$ changes the sign of the surd term: $\mathbf{\frac{21-7\sqrt{5}}{2}}$.

**We Do**

**(a)** $\frac{15(3+\sqrt{3})}{(3-\sqrt{3})(3+\sqrt{3})} = \frac{15(3+\sqrt{3})}{9-3} = \frac{15(3+\sqrt{3})}{6} = \frac{5(3+\sqrt{3})}{2} = \mathbf{\frac{15+5\sqrt{3}}{2}}$ ($a=15,\ b=5$).

**(b)** $\mathbf{\frac{15-5\sqrt{3}}{2}}$.

**You Do 1**

**(a)** $\frac{10(4+\sqrt{10})}{(4-\sqrt{10})(4+\sqrt{10})} = \frac{10(4+\sqrt{10})}{16-10} = \frac{10(4+\sqrt{10})}{6} = \frac{5(4+\sqrt{10})}{3} = \mathbf{\frac{20+5\sqrt{10}}{3}}$ ($a=20,\ b=5$).

**(b)** $\mathbf{\frac{20-5\sqrt{10}}{3}}$.

**You Do 2**

**(a)** $\frac{12(5+\sqrt{17})}{(5-\sqrt{17})(5+\sqrt{17})} = \frac{12(5+\sqrt{17})}{25-17} = \frac{12(5+\sqrt{17})}{8} = \frac{3(5+\sqrt{17})}{2} = \mathbf{\frac{15+3\sqrt{17}}{2}}$ ($a=15,\ b=3$).

**(b)** $\mathbf{\frac{15-3\sqrt{17}}{2}}$.

**You Do 3**

**(a)** $\frac{(3+\sqrt{2})(4+\sqrt{2})}{(4-\sqrt{2})(4+\sqrt{2})} = \frac{12+3\sqrt{2}+4\sqrt{2}+2}{16-2} = \frac{14+7\sqrt{2}}{14} = \mathbf{\frac{2+\sqrt{2}}{2}}$.

**(b)** Rationalise each term separately:

$\frac{10}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$.

$\frac{6(2+\sqrt{5})}{(2-\sqrt{5})(2+\sqrt{5})} = \frac{12+6\sqrt{5}}{4-5} = \frac{12+6\sqrt{5}}{-1} = -12-6\sqrt{5}$.

Add the two results: $2\sqrt{5}-12-6\sqrt{5} = \mathbf{-12-4\sqrt{5}}$.

**Challenge 1**

1. Simplify $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$.
2. Multiply numerator and denominator by $(3 + \sqrt{3})$:

$$\frac{(6 + 2\sqrt{3})(3 + \sqrt{3})}{(3 - \sqrt{3})(3 + \sqrt{3})} = \frac{18 + 6\sqrt{3} + 6\sqrt{3} + 6}{9 - 3} = \frac{24 + 12\sqrt{3}}{6} = 4 + 2\sqrt{3}.$$

Therefore, **a** $= 4$ and **b** $= 2$.

**Challenge 2**

**(a)** Length $= \dfrac{\text{Area}}{\text{Width}} = \dfrac{11 + 5\sqrt{3}}{2 + \sqrt{3}}$.

Multiply numerator and denominator by $(2 - \sqrt{3})$:

$$\frac{(11 + 5\sqrt{3})(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})} = \frac{22 - 11\sqrt{3} + 10\sqrt{3} - 15}{4 - 3} = \frac{7 - \sqrt{3}}{1} = (\mathbf{7} - \sqrt{\mathbf{3}})\text{ cm}.$$

**(b)** Perimeter $= 2(\text{Length} + \text{Width}) = 2\big((7 - \sqrt{3}) + (2 + \sqrt{3})\big) = 2(9) =$ **18** cm.

**Extreme Challenge**

**(a)** Multiply numerator and denominator by the conjugate $(\sqrt{n+1} - \sqrt{n})$:

$$\frac{1(\sqrt{n+1} - \sqrt{n})}{(\sqrt{n+1} + \sqrt{n})(\sqrt{n+1} - \sqrt{n})} = \frac{\sqrt{n+1} - \sqrt{n}}{(n+1) - n} = \frac{\sqrt{n+1} - \sqrt{n}}{1} = \sqrt{n+1} - \sqrt{n}.$$ *(Shown)*

**(b)** Apply the result from part (a) to every term in the sum (noting $1 = \sqrt{1}$):

$(\sqrt{2} - \sqrt{1}) + (\sqrt{3} - \sqrt{2}) + (\sqrt{4} - \sqrt{3}) + \cdots + (\sqrt{36} - \sqrt{35})$.

All intermediate terms cancel out (a telescoping sum), leaving:

$\sqrt{36} - \sqrt{1} = 6 - 1 =$ **5**.