Similar Solids: Surface Area & Volume

Practice Worksheet (Problem Solving)

I Do

Solids A and B are mathematically similar.
Their total surface areas are $40\,\text{cm}^2$ and $90\,\text{cm}^2$ respectively.
The volume of A is $120\,\text{cm}^3$. Calculate the volume of B.

.....cm^3

(Total for Question is 4 marks)

We Do

Solids A and B are mathematically similar.
Their total surface areas are $25\,\text{cm}^2$ and $64\,\text{cm}^2$ respectively.
The volume of A is $250\,\text{cm}^3$. Calculate the volume of B.

.....cm^3

(Total for Question is 3 marks)

You Do 1

Solids P and Q are mathematically similar.
Their total surface areas are $32\,\text{cm}^2$ and $72\,\text{cm}^2$ respectively.
The volume of Q is $540\,\text{cm}^3$. Calculate the volume of P.

.....cm^3

(Total for Question is 3 marks)

You Do 2

Cone A and Cone B are mathematically similar.
The volume of Cone A is $54\,\text{cm}^3$ and the volume of Cone B is $128\,\text{cm}^3$.
The total surface area of Cone A is $45\,\text{cm}^2$.

Calculate the total surface area of Cone B.

.....cm^2

(Total for Question is 3 marks)

You Do 3

Cylinder X and Cylinder Y are mathematically similar.
The volume of Cylinder X is $160\,\text{cm}^3$ and the volume of Cylinder Y is $540\,\text{cm}^3$.
The total surface area of Cylinder Y is $315\,\text{cm}^2$.

Calculate the total surface area of Cylinder X.

.....cm^2

(Total for Question is 3 marks)

Challenge 1

Solids A, B, and C are mathematically similar.

- The ratio of the height of A to the height of B is $2 : 3$.
- The ratio of the total surface area of B to the total surface area of C is $9 : 16$.

(a) Find the ratio of the volume of A to the volume of C.
Give your answer in its simplest form.

..... *(2 marks)*

(b) Given that the volume of C is $448\,\text{cm}^3$, work out the volume of B.

.....cm^3 *(2 marks)*

(Total for Question is 4 marks)

Challenge 2

A large cone has a perpendicular height of $15\,\text{cm}$ and a volume of $625\,\text{cm}^3$.

A smaller cone of perpendicular height $6\,\text{cm}$ is cut off the top of the large cone, parallel to its base, leaving a frustum.

Calculate the volume of the frustum.
Show all of your working clearly.

.....cm^3

(Total for Question is 4 marks)

Extreme Challenge

Solids A and B are mathematically similar.
The total surface area of solid B is 125% greater than the total surface area of solid A.

The difference between the volume of solid B and the volume of solid A is $380\,\text{cm}^3$.

(a) Work out the volume of solid A.
You must show all of your working clearly.

.....cm^3 *(4 marks)*

(b) Calculate the percentage increase from the volume of solid A to the volume of solid B.

.....% *(1 mark)*

(Total for Question is 5 marks)

Worked Answers

I Do

1. Area scale factor (A to B) $= \frac{90}{40} = \frac{9}{4} = 2.25$.
2. Length scale factor $= \sqrt{\frac{9}{4}} = \frac{3}{2} = 1.5$.
3. Volume scale factor $= \left(\frac{3}{2}\right)^3 = \frac{27}{8} = 3.375$.
4. Volume of B $= 120 \times \frac{27}{8} = 15 \times 27 =$ **405** cm^3.

We Do

Area ratio $= 25 : 64 \implies$ Length ratio $= \sqrt{25} : \sqrt{64} = 5 : 8$.
Volume ratio $= 5^3 : 8^3 = 125 : 512$.
Volume of B $= 250 \times \frac{512}{125} = 2 \times 512 =$ **1024** cm^3.

You Do 1

Area ratio (P to Q) $= 32 : 72 = 4 : 9 \implies$ Length ratio $= 2 : 3$.
Volume ratio $= 2^3 : 3^3 = 8 : 27$.
Volume of P $= 540 \times \frac{8}{27} = 20 \times 8 =$ **160** cm^3.

You Do 2

Volume ratio (A to B) $= 54 : 128 = 27 : 64 \implies$ Length ratio $= \sqrt[3]{27} : \sqrt[3]{64} = 3 : 4$.
Area ratio $= 3^2 : 4^2 = 9 : 16$.
Total surface area of Cone B $= 45 \times \frac{16}{9} = 5 \times 16 =$ **80** cm^2.

You Do 3

Volume ratio (X to Y) $= 160 : 540 = 8 : 27 \implies$ Length ratio $= \sqrt[3]{8} : \sqrt[3]{27} = 2 : 3$.
Area ratio $= 2^2 : 3^2 = 4 : 9$.
Total surface area of Cylinder X $= 315 \times \frac{4}{9} = 35 \times 4 =$ **140** cm^2.

Challenge 1

(a) 1. Surface area ratio $B : C = 9 : 16 \implies$ Height ratio $B : C = \sqrt{9} : \sqrt{16} = 3 : 4$.
2. Combine with Height ratio $A : B = 2 : 3$ to get Height ratio $A : B : C = 2 : 3 : 4$.
3. Cube to find Volume ratio $A : C = 2^3 : 4^3 = 8 : 64 =$ **1** : **8**.

(b) Volume ratio $B : C = 3^3 : 4^3 = 27 : 64$.
Volume of B $= 448 \times \frac{27}{64} = 7 \times 27 =$ **189** cm^3.

Challenge 2

1. The smaller cone cut off the top is mathematically similar to the large cone.
Length ratio (small to large) $= 6 : 15 = 2 : 5$.
2. Volume ratio (small to large) $= 2^3 : 5^3 = 8 : 125$.
3. Volume of the smaller cone $= 625 \times \frac{8}{125} = 5 \times 8 = 40$ cm^3.
4. Subtract from the large cone to find the frustum:
Volume of frustum $= 625 - 40 =$ **585** cm^3.

Extreme Challenge

(a) 1. A 125% increase means the area scale factor from A to B is:
$1 + 1.25 = 2.25 = \frac{9}{4}$.
2. Find the length and volume scale factors:
Length scale factor $= \sqrt{\frac{9}{4}} = \frac{3}{2}$.
Volume scale factor $= \left(\frac{3}{2}\right)^3 = \frac{27}{8}$ (so Volume ratio $A : B = 8 : 27$).
3. The difference in volume is $27 - 8 = 19$ parts:
19 parts $= 380$ cm^3 $\implies 1$ part $= \frac{380}{19} = 20$ cm^3.
4. Volume of solid A $= 8 \times 20 =$ **160** cm^3.

(b) Volume scale factor $= \frac{27}{8} = 3.375$.
Percentage increase $= (3.375 - 1) \times 100\% =$ **237.5**%.