Kinematics & Differentiation

Practice Worksheet (Problem Solving)

I Do

A particle moves along a straight line. Its displacement s metres from a fixed point O, at time t seconds, is given by

$$s = 2t^3 - 9t^2 + 12t + 5, \text{ where } t \geq 0.$$

(a) Find an expression for the velocity v of the particle, in m/s.

$v =$ ..... *(2 marks)*

(b) Find the times when the particle is instantaneously at rest. Show clear algebraic working.

$t =$ ..... and $t =$ ..... *(3 marks)*

(Total for Question is 5 marks)

We Do

A particle moves along a straight line. Its displacement s metres from a fixed point O, at time t seconds, is given by

$$s = t^3 - 6t^2 + 9t + 4, \text{ where } t \geq 0.$$

(a) Find an expression for the velocity v of the particle, in m/s.

$v =$ ..... *(2 marks)*

(b) Find the times when the particle is instantaneously at rest. Show clear algebraic working.

$t =$ ..... and $t =$ ..... *(3 marks)*

(Total for Question is 5 marks)

You Do 1

A particle moves along a straight line. Its displacement s metres from a fixed point O, at time t seconds, is given by

$$s = 2t^3 - 15t^2 + 24t - 3, \text{ where } t \geq 0.$$

(a) Find an expression for the velocity v of the particle, in m/s.

$v =$ ..... *(2 marks)*

(b) Find the times when the particle is instantaneously at rest. Show clear algebraic working.

$t =$ ..... and $t =$ ..... *(3 marks)*

(Total for Question is 5 marks)

You Do 2

A particle moves along a straight line. Its displacement s metres from a fixed point O, at time t seconds, is given by

$$s = t^3 - 9t^2 + 15t + 10, \text{ where } t \geq 0.$$

(a) Find an expression for the velocity v of the particle, in m/s.

$v =$ ..... *(2 marks)*

(b) Find the times when the particle is instantaneously at rest. Show clear algebraic working.

$t =$ ..... and $t =$ ..... *(3 marks)*

(c) Find the acceleration of the particle when $t = 4$.

.....m/s^2 *(2 marks)*

(Total for Question is 7 marks)

You Do 3

A particle moves along a straight line. Its displacement s metres from a fixed point O, at time t seconds, is given by

$$s = \frac{1}{3}t^3 - 2t^2 - 5t + 8, \text{ where } t \geq 0.$$

(a) Find the time when the particle is instantaneously at rest. Show clear algebraic working.

$t =$ .....s *(3 marks)*

(b) Find the acceleration of the particle at the instant when it is at rest.

.....m/s^2 *(2 marks)*

(Total for Question is 5 marks)

Challenge 1

A particle moves along a straight line. Its displacement s metres from a fixed point O, at time t seconds, is given by

$$s = 2t^3 - 12t^2 + 18t + 1, \text{ where } t \geq 0.$$

(a) Find the distance travelled by the particle between the two times when it is instantaneously at rest.
Show all of your working clearly.

.....m *(4 marks)*

(b) Find the minimum velocity of the particle.

.....m/s *(2 marks)*

(Total for Question is 6 marks)

Challenge 2

A particle moves along a straight line. Its displacement s metres from a fixed point O, at time t seconds, is given by

$$s = t^3 + pt^2 + qt + 4, \text{ where } t \geq 0$$

and p and q are constants.

The particle is instantaneously at rest when $t = 2$ and when $t = 4$.

(a) Find the value of p and the value of q.
Show all of your algebraic working clearly.

$p =$ ..... $q =$ ..... *(4 marks)*

(b) Find the acceleration of the particle when $t = 5$.

.....m/s^2 *(2 marks)*

(Total for Question is 6 marks)

Extreme Challenge

A particle moves along a straight line. Its displacement s metres from a fixed point O, at time t seconds, is given by

$$s = t^3 - 3t^2 - 9t + 10, \text{ where } t \geq 0.$$

(a) Show that the particle is instantaneously at rest only once for $t \geq 0$, and find this time.

$t =$ .....s *(3 marks)*

(b) Calculate the **total distance** travelled by the particle between $t = 0$ and $t = 4$.
You must show all of your working clearly.

.....m *(3 marks)*

(Total for Question is 6 marks)

Worked Answers

I Do
(a) Differentiate s with respect to t: $\mathbf{v = 6t^2 - 18t + 12}$.
(b) Set $v = 0$:
$6t^2 - 18t + 12 = 0$
Divide by 6: $t^2 - 3t + 2 = 0 \implies (t-1)(t-2) = 0$.
$\mathbf{t = 1}$ **and** $\mathbf{t = 2}$.

We Do
(a) $\mathbf{v = 3t^2 - 12t + 9}$.
(b) $3t^2 - 12t + 9 = 0 \implies t^2 - 4t + 3 = 0 \implies (t-1)(t-3) = 0$.
$\mathbf{t = 1}$ **and** $\mathbf{t = 3}$.

You Do 1
(a) $\mathbf{v = 6t^2 - 30t + 24}$.
(b) $6t^2 - 30t + 24 = 0 \implies t^2 - 5t + 4 = 0 \implies (t-1)(t-4) = 0$.
$\mathbf{t = 1}$ **and** $\mathbf{t = 4}$.

You Do 2
(a) $\mathbf{v = 3t^2 - 18t + 15}$.
(b) $3t^2 - 18t + 15 = 0 \implies t^2 - 6t + 5 = 0 \implies (t-1)(t-5) = 0 \implies$ $\mathbf{t = 1}$ **and** $\mathbf{t = 5}$.
(c) Differentiate v: $a = \dfrac{dv}{dt} = 6t - 18$.
When $t = 4$: $a = 6(4) - 18 =$ **6** m/s^2.

You Do 3
(a) $v = t^2 - 4t - 5 = 0 \implies (t-5)(t+1) = 0 \implies t = 5$ or $t = -1$.
Since $t \geq 0$, $\mathbf{t = 5}$ s.
(b) $a = \dfrac{dv}{dt} = 2t - 4$.
When $t = 5$: $a = 2(5) - 4 =$ **6** m/s^2.

Challenge 1

(a) 1. Find when $v = 0$:
$v = 6t^2 - 24t + 18 = 0 \implies t^2 - 4t + 3 = 0 \implies (t-1)(t-3) = 0 \implies t = 1$ and $t = 3$.
2. Find the displacement at each time:
When $t = 1$: $s = 2(1)^3 - 12(1)^2 + 18(1) + 1 = 9$ m.
When $t = 3$: $s = 2(3)^3 - 12(3)^2 + 18(3) + 1 = 54 - 108 + 54 + 1 = 1$ m.
3. Distance travelled between $t = 1$ and $t = 3$: $|1 - 9| =$ **8** m.

(b) Minimum velocity occurs when acceleration $a = 0$:
$a = 12t - 24 = 0 \implies t = 2$.
Substitute $t = 2$ into v: $v = 6(2)^2 - 24(2) + 18 = 24 - 48 + 18 = -$**6** m/s.

Challenge 2

(a) Differentiate s to find v: $v = 3t^2 + 2pt + q$.
Since $v = 0$ at $t = 2$ and $t = 4$:
$v = 3(t-2)(t-4) = 3(t^2 - 6t + 8) = 3t^2 - 18t + 24$.
Comparing coefficients:
$2p = -18 \implies$ **p** $= -$**9**, and **q** $=$ **24**.
(Alternatively, form and solve the simultaneous equations $4p + q = -12$ and $8p + q = -48$.)

(b) $a = \dfrac{dv}{dt} = 6t - 18$.
When $t = 5$: $a = 6(5) - 18 =$ **12** m/s^2.

Extreme Challenge

(a) Differentiate s: $v = 3t^2 - 6t - 9$.
Set $v = 0$: $3(t^2 - 2t - 3) = 0 \implies (t-3)(t+1) = 0 \implies t = 3$ or $t = -1$.
Since $t \geq 0$, $t = -1$ is rejected, so the particle is instantaneously at rest only once, at **t** $=$ **3** s.

(b) Because the particle changes direction at $t = 3$, split the journey into two parts ($t = 0$ to $t = 3$, and $t = 3$ to $t = 4$):
- At $t = 0$: $s = 10$ m.
- At $t = 3$: $s = (3)^3 - 3(3)^2 - 9(3) + 10 = 27 - 27 - 27 + 10 = -17$ m.
- At $t = 4$: $s = (4)^3 - 3(4)^2 - 9(4) + 10 = 64 - 48 - 36 + 10 = -10$ m.
Distance from $t = 0$ to $t = 3$: $|-17 - 10| = 27$ m.
Distance from $t = 3$ to $t = 4$: $|-10 - (-17)| = 7$ m.
Total distance travelled $= 27 + 7 =$ **34** m.