# 3D Trigonometry & Pythagoras

*Practice Worksheet (Problem Solving)*

## I Do

$ABCDEFGH$ is a cuboid. $ABCD$ is the base, with $E$, $F$, $G$ and $H$ vertically above $A$, $B$, $C$ and $D$.
$AB = 10$ cm, $BC = 6$ cm and $CG = 5$ cm.
Calculate the angle between $BH$ and plane $ABCD$. Give your answer to $1$ decimal place.

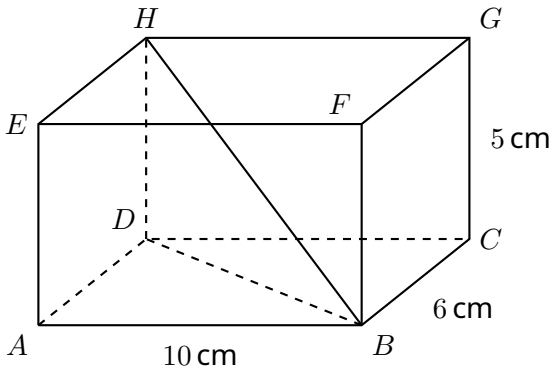


Diagram not to scale

.....°

**(Total for Question is 4 marks)**

## We Do

$ABCDEFGH$ is a cuboid. $ABCD$ is the base, with $E$, $F$, $G$ and $H$ vertically above $A$, $B$, $C$ and $D$.
$AB = 12$ cm, $BC = 5$ cm and $CG = 8$ cm.
Calculate the angle between $AG$ and plane $ABCD$. Give your answer to $1$ decimal place.

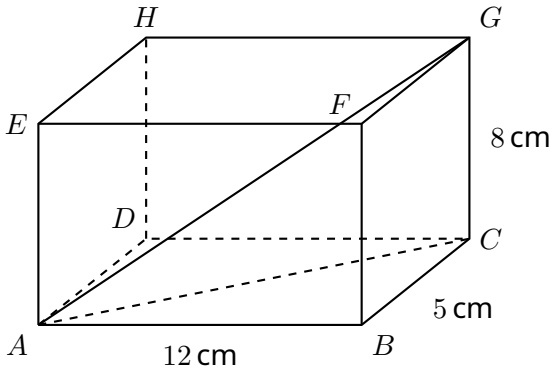


Diagram not to scale

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**(Total for Question is 4 marks)**

## You Do 1

$ABCDEFGH$ is a cuboid. $ABCD$ is the base, with $E$, $F$, $G$ and $H$ vertically above $A$, $B$, $C$ and $D$.
$AB = 8$ cm, $BC = 6$ cm and $CG = 7$ cm.
Calculate the angle between $CE$ and plane $ABCD$. Give your answer to $1$ decimal place.

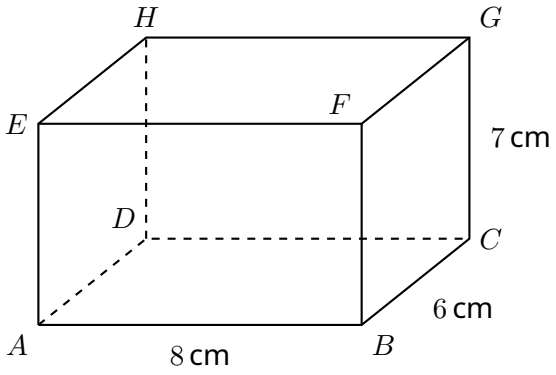


Diagram not to scale

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**(Total for Question is 4 marks)**

## You Do 2

$ABCDEFGH$ is a cuboid. $ABCD$ is the base, with $E$, $F$, $G$ and $H$ vertically above $A$, $B$, $C$ and $D$.
$AB = 9$ cm, $BC = 8$ cm and $CG = 6$ cm.
Calculate the angle between $AG$ and plane $BCGF$. Give your answer to $1$ decimal place.

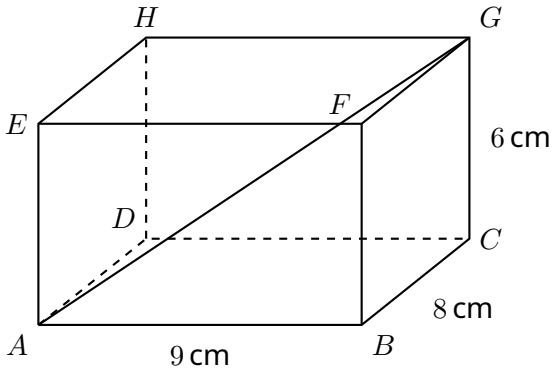


Diagram not to scale

.....°

**(Total for Question is 4 marks)**

## You Do 3

$ABCDEFGH$ is a cuboid. $ABCD$ is the base, with $E$, $F$, $G$ and $H$ vertically above $A$, $B$, $C$ and $D$.
$AB = 14$ cm and $BC = 8$ cm.
The angle between $BH$ and plane $ABCD$ is $28°$.
Calculate the height $CG$ of the cuboid. Give your answer to $1$ decimal place.

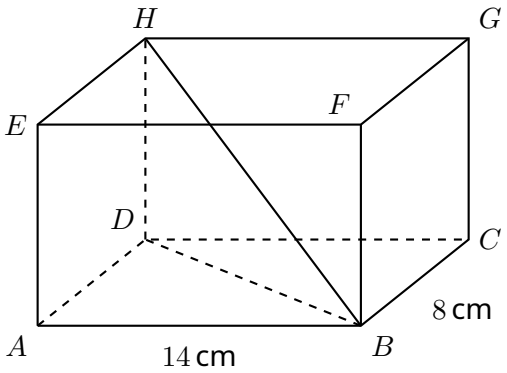


Diagram not to scale

.....cm

**(Total for Question is 4 marks)**

## Challenge 1

$VABCD$ is a square-based pyramid.
The base $ABCD$ is a square of side length $10$ cm.
The vertex $V$ is vertically above the centre $O$ of the base, and $VO = 12$ cm.

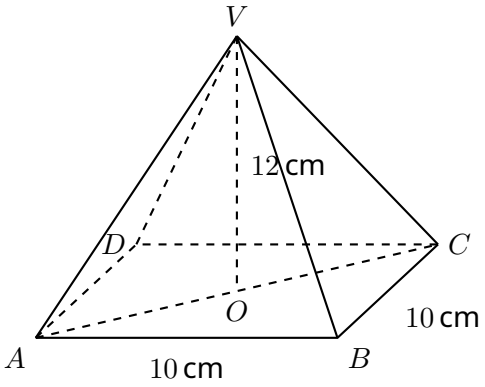


Diagram not to scale

Calculate the angle between the sloping edge $VA$ and the base $ABCD$.
Give your answer correct to $1$ decimal place.

.....°

**(Total for Question is 4 marks)**

## Challenge 2

$ABCDEFGH$ is a cuboid. $ABCD$ is the base, with $E$, $F$, $G$ and $H$ vertically above $A$, $B$, $C$ and $D$.
$AB = 12$ cm, $BC = 8$ cm and $CG = 11$ cm.
$M$ is the midpoint of $EF$.

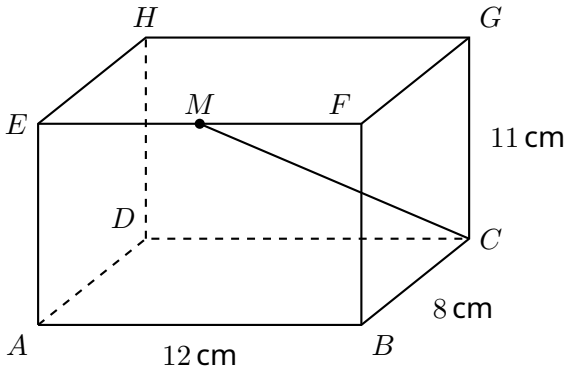


Diagram not to scale

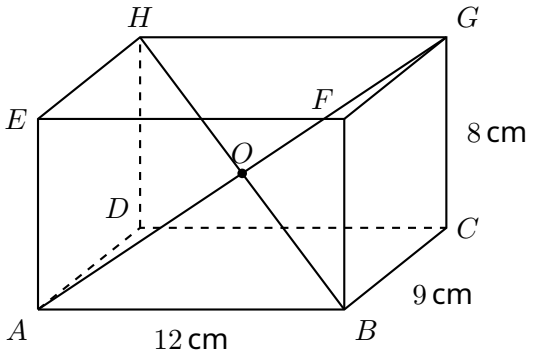
Calculate the angle between $CM$ and plane $ABCD$.
Give your answer correct to $1$ decimal place.

.....°

**(Total for Question is 4 marks)**

## Extreme Challenge

$ABCDEFGH$ is a cuboid with $AB = 12$ cm, $BC = 9$ cm and $CG = 8$ cm.
The space diagonals $AG$ and $BH$ intersect at their midpoints at point $O$.




**(a)** Calculate the exact length of $AG$.

.....cm *(2 marks)*

**(b)** Calculate the size of angle $AOB$.
Give your answer correct to $1$ decimal place.

.....° *(3 marks)*

**(Total for Question is 5 marks)**

# Worked Answers

**I Do**

1. Find diagonal $BD$ on the base $ABCD$ using Pythagoras' theorem:
$BD^2 = 10^2 + 6^2 = 100 + 36 = 136 \implies BD = \sqrt{136}$ cm ($\approx 11.662$ cm).
2. In right-angled triangle $BDH$, $DH = CG = 5$ cm:
$\tan(\angle HBD) = \dfrac{5}{\sqrt{136}} \implies \angle HBD = \tan^{-1}\left(\dfrac{5}{\sqrt{136}}\right) = 23.207\ldots^\circ \approx \mathbf{23.2}^\circ$.

**We Do**

1. $AC = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$ cm.
2. In right-angled triangle $ACG$:
$\tan(\angle GAC) = \dfrac{8}{13} \implies \angle GAC = \tan^{-1}\left(\dfrac{8}{13}\right) = 31.607\ldots^\circ \approx \mathbf{31.6}^\circ$.

**You Do 1**

1. $AC = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$ cm.
2. In right-angled triangle $CAE$, $AE = CG = 7$ cm:
$\tan(\angle ECA) = \dfrac{7}{10} = 0.7 \implies \angle ECA = \tan^{-1}(0.7) = 34.992\ldots^\circ \approx \mathbf{35.0}^\circ$.

**You Do 2**

1. Since $AB$ is perpendicular to plane $BCGF$, the projection of $AG$ onto plane $BCGF$ is $BG$.
$BG = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$ cm.
2. In right-angled triangle $ABG$:
$\tan(\angle AGB) = \dfrac{AB}{BG} = \dfrac{9}{10} = 0.9 \implies \angle AGB = \tan^{-1}(0.9) = 41.987\ldots^\circ \approx \mathbf{42.0}^\circ$.

**You Do 3**

1. $BD = \sqrt{14^2 + 8^2} = \sqrt{196 + 64} = \sqrt{260}$ cm ($\approx 16.125$ cm).
2. In right-angled triangle $BDH$:
$\tan(28^\circ) = \dfrac{DH}{\sqrt{260}} \implies DH = \sqrt{260} \times \tan(28^\circ) = 8.573\cdots \approx \mathbf{8.6}$ cm.
Since $CG = DH$, **CG** $= \mathbf{8.6}$ cm.

**Challenge 1**

1. Find the diagonal $AC$ of the square base $ABCD$:
$AC = \sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2}$ cm.
2. Since $O$ is the centre of the base, $AO = \frac{1}{2}AC = 5\sqrt{2}$ cm ($\approx 7.071$ cm).
3. In right-angled triangle $VOA$:
$\tan(\angle VAO) = \frac{12}{5\sqrt{2}} \implies \angle VAO = \tan^{-1}\left(\frac{12}{5\sqrt{2}}\right) = 59.491\ldots^\circ \approx \mathbf{59.5}^\circ$.

**Challenge 2**

1. Let $N$ be the midpoint of $AB$, vertically below $M$. Then $MN = 11$ cm and $NB = \frac{12}{2} = 6$ cm.
2. Find $NC$ on the base $ABCD$ using right-angled triangle $NBC$:
$NC = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$ cm.
3. In right-angled triangle $MNC$:
$\tan(\angle MCN) = \frac{11}{10} = 1.1 \implies \angle MCN = \tan^{-1}(1.1) = 47.726\ldots^\circ \approx \mathbf{47.7}^\circ$.

**Extreme Challenge**

**(a)** Use 3D Pythagoras' theorem:
$AG = \sqrt{12^2 + 9^2 + 8^2} = \sqrt{144 + 81 + 64} = \sqrt{289} = \mathbf{17}$ cm.

**(b)** Since $AG = BH = 17$ cm and $O$ is the midpoint of both diagonals:
$OA = OB = \frac{17}{2} = 8.5$ cm, and $AB = 12$ cm.
Use the cosine rule in triangle $AOB$:
$\cos(\angle AOB) = \frac{8.5^2 + 8.5^2 - 12^2}{2 \times 8.5 \times 8.5} = \frac{72.25 + 72.25 - 144}{144.5} = \frac{0.5}{144.5} = \frac{1}{289}$.
$\angle AOB = \cos^{-1}\left(\frac{1}{289}\right) = 89.801\ldots^\circ \approx \mathbf{89.8}^\circ$.