Algebraic Probability

Practice Worksheet (Problem Solving)

I Do

A bag contains only red and blue counters. Exactly 5 counters are red.
Two counters are taken at random without replacement.
The probability that both are red is

$$\frac{5}{14}$$

Work out the total number of counters. Show clear algebraic working.

.....

(Total for Question is 5 marks)

We Do

A bag contains only green and yellow counters. Exactly 6 counters are green.
Two counters are taken at random without replacement.
The probability that both are green is

$$\frac{1}{3}$$

Work out the total number of counters. Show clear algebraic working.

.....

(Total for Question is 4 marks)

You Do 1

A bag contains only red and white counters. Exactly 4 counters are red.
Two counters are taken at random without replacement.
The probability that both are red is

$$\frac{1}{6}$$

Work out the total number of counters. Show clear algebraic working.

.....

(Total for Question is 4 marks)

You Do 2

A bag contains only black and blue counters. Exactly 7 counters are black.
Two counters are taken at random without replacement.
The probability that both are black is

$$\frac{1}{5}$$

Work out the number of **blue** counters in the bag. Show clear algebraic working.

.....

(Total for Question is 4 marks)

You Do 3

A bag contains only red and blue counters.
There are 4 red counters and n blue counters in the bag.
Two counters are taken at random without replacement.
The probability that both counters are blue is

$$\frac{1}{3}$$

Work out the value of n. Show clear algebraic working.

$n =$ .....

(Total for Question is 5 marks)

Challenge 1

A bag contains n red counters and 5 blue counters.
Two counters are taken at random without replacement.
The probability that both counters are the **same colour** is

$$\frac{4}{9}$$

(a) Show that $n^2 - 9n + 20 = 0$.

(3 marks)

(b) Hence, find the two possible values of n.

$n =$ ..... or $n =$ ..... *(2 marks)*

(Total for Question is 5 marks)

Challenge 2

A bag contains only red, blue, and green counters.
There are x red counters, 3 blue counters, and $(x + 1)$ green counters.
Two counters are taken at random without replacement.
The probability that both counters are green is

$$\frac{2}{15}$$

Work out the total number of counters in the bag.
You must show all of your algebraic working clearly.

.....

(Total for Question is 5 marks)

Extreme Challenge

A bag contains n counters in total, of which 6 are red and the rest are blue.
Two counters are taken at random without replacement.
The probability that **at least one** of the two counters is red is

$$\frac{13}{15}$$

(a) Show that $13n^2 - 193n + 630 = 0$, and hence find the value of n.
You must show all of your algebraic working clearly.

$n =$ ..... *(4 marks)*

(b) Three counters are taken at random without replacement from the original bag.
Work out the probability that all three counters are red.
Give your answer as a fraction in its simplest form.

..... *(2 marks)*

(Total for Question is 6 marks)

Worked Answers

I Do

Let the total number of counters be n.

$P(\text{both red}) = \frac{5}{n} \times \frac{4}{n-1} = \frac{20}{n(n-1)} = \frac{5}{14}$.

Divide both numerators by 5: $\frac{4}{n(n-1)} = \frac{1}{14} \implies n(n-1) = 56$.

$n^2 - n - 56 = 0 \implies (n-8)(n+7) = 0 \implies n = 8$ or $n = -7$.

Since $n > 0$, there are **8 counters in total**.

We Do

$\frac{6}{n} \times \frac{5}{n-1} = \frac{30}{n(n-1)} = \frac{1}{3} \implies n(n-1) = 90$.

$n^2 - n - 90 = 0 \implies (n-10)(n+9) = 0 \implies$ **n = 10**.

You Do 1

$\frac{4}{n} \times \frac{3}{n-1} = \frac{12}{n(n-1)} = \frac{1}{6} \implies n(n-1) = 72$.

$n^2 - n - 72 = 0 \implies (n-9)(n+8) = 0 \implies$ **n = 9**.

You Do 2

$\frac{7}{n} \times \frac{6}{n-1} = \frac{42}{n(n-1)} = \frac{1}{5} \implies n(n-1) = 210$.

$n^2 - n - 210 = 0 \implies (n-15)(n+14) = 0 \implies n = 15$ counters in total.

Number of blue counters $= 15 - 7 =$ **8**.

You Do 3

Total counters $= n + 4$.

$P(\text{both blue}) = \frac{n}{n+4} \times \frac{n-1}{n+3} = \frac{1}{3}$.

Cross-multiply: $3n(n-1) = (n+4)(n+3) \implies 3n^2 - 3n = n^2 + 7n + 12$.

$2n^2 - 10n - 12 = 0 \implies n^2 - 5n - 6 = 0 \implies (n-6)(n+1) = 0$.

Since $n > 0$, **n = 6**.

Challenge 1

(a) Total counters $= n + 5$.

$P(\text{same colour}) = P(\text{RR}) + P(\text{BB}) = \dfrac{n(n-1)}{(n+5)(n+4)} + \dfrac{5 \times 4}{(n+5)(n+4)} = \dfrac{4}{9}$.

$\dfrac{n^2 - n + 20}{n^2 + 9n + 20} = \dfrac{4}{9}$.

Cross-multiply:

$9(n^2 - n + 20) = 4(n^2 + 9n + 20)$

$9n^2 - 9n + 180 = 4n^2 + 36n + 80$

$5n^2 - 45n + 100 = 0 \implies n^2 - 9n + 20 = 0$. ***(Shown)***

(b) Factorise: $(n-4)(n-5) = 0 \implies$ **n = 4 or n = 5**.

Challenge 2

1. Total counters $= x + 3 + (x + 1) = 2x + 4$.
2. $P(\text{both green}) = \dfrac{x+1}{2x+4} \times \dfrac{x}{2x+3} = \dfrac{2}{15}$.
3. Cross-multiply:

$15x(x+1) = 2(2x+4)(2x+3)$

$15x^2 + 15x = 2(4x^2 + 14x + 12) = 8x^2 + 28x + 24$

$7x^2 - 13x - 24 = 0 \implies (7x+8)(x-3) = 0$.

4. Since x is a positive integer, $x = 3$.
5. Total counters $= 2(3) + 4 =$ **10**.

Extreme Challenge

(a) 1. Use complementary probability:

$P(\text{both blue}) = 1 - P(\text{at least one red}) = 1 - \dfrac{13}{15} = \dfrac{2}{15}$.

2. There are $(n-6)$ blue counters out of n total:

$\dfrac{n-6}{n} \times \dfrac{n-7}{n-1} = \dfrac{2}{15}$.

3. Cross-multiply:

$15(n^2 - 13n + 42) = 2(n^2 - n)$

$15n^2 - 195n + 630 = 2n^2 - 2n$

$13n^2 - 193n + 630 = 0$. ***(Shown)***

4. Factorise: $(13n - 63)(n - 10) = 0 \implies n = \dfrac{63}{13}$ or $n = 10$.

Since n must be an integer, **n** $= 10$.

(b) With 6 red counters out of 10:

$P(\text{RRR}) = \dfrac{6}{10} \times \dfrac{5}{9} \times \dfrac{4}{8} = \dfrac{120}{720} = \mathbf{\dfrac{1}{6}}$.