Cosine Rule & Quadratics

Practice Worksheet (Problem Solving)

I Do

In triangle ABC, $AB = x$ cm, $BC = (x + 2)$ cm, $AC = 7$ cm and angle $ABC = 60°$.
Find the exact value of x. Show clear algebraic working.

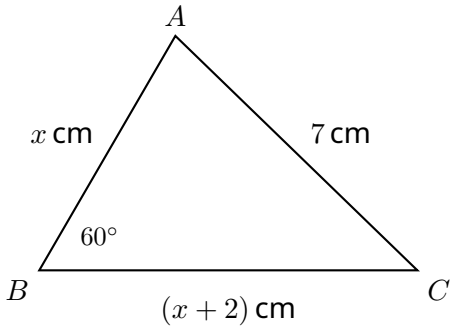


Diagram not to scale

$x =$ .....

(Total for Question is 5 marks)

We Do

In triangle ABC, $AB = x$ cm, $BC = (x + 3)$ cm, $AC = 7$ cm and angle $ABC = 60°$.
Find the exact value of x. Show clear algebraic working.

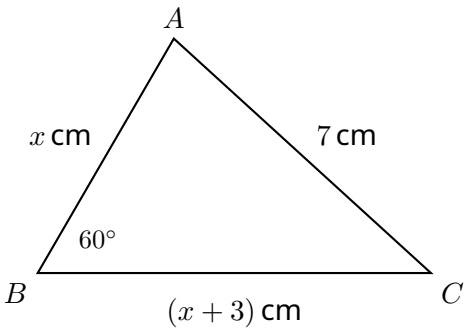


Diagram not to scale

$x =$ .....

(Total for Question is 5 marks)

You Do 1

In triangle ABC, $AB = x$ cm, $BC = (x + 4)$ cm, $AC = 8$ cm and angle $ABC = 60°$.
Find the exact value of x. Give your answer in its simplest surd form.

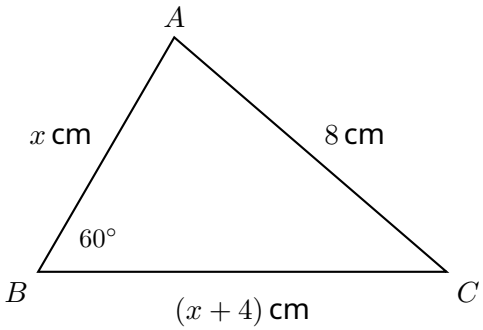


Diagram not to scale

$x =$ .....

(Total for Question is 5 marks)

You Do 2

In triangle ABC, $AB = x$ cm, $BC = (x + 2)$ cm, $AC = 7$ cm and angle $ABC = 120°$.
Find the value of x. Show clear algebraic working.

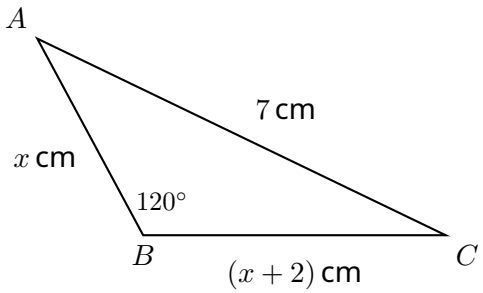


Diagram not to scale

$x =$ .....

(Total for Question is 5 marks)

You Do 3

In triangle PQR, $PQ = x$ cm, $QR = (2x - 1)$ cm, $PR = 7$ cm and angle $PQR = 120°$.
Find the value of x. Show clear algebraic working.

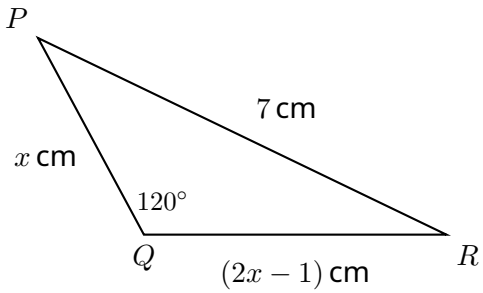


Diagram not to scale

$x =$ .....

(Total for Question is 5 marks)

Challenge 1

In triangle ABC, $AB = (x + 1)$ cm, $BC = (x + 3)$ cm, $AC = 2x$ cm and angle $ABC = 60°$.

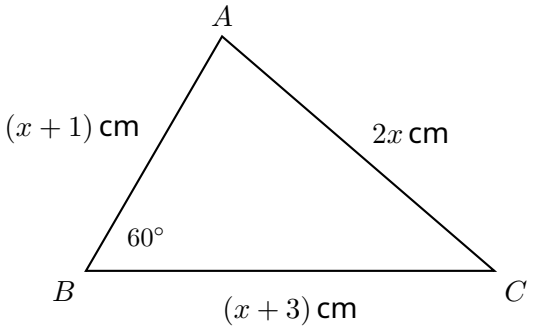


Diagram not to scale

Show that $3x^2 - 4x - 7 = 0$, and hence find the exact value of x.
Show all of your algebraic working clearly.

$x =$ .....

(Total for Question is 5 marks)

Challenge 2

In triangle PQR, $PQ = x$ cm, $QR = (x + 5)$ cm, $PR = 7$ cm and angle $PQR = 60°$.

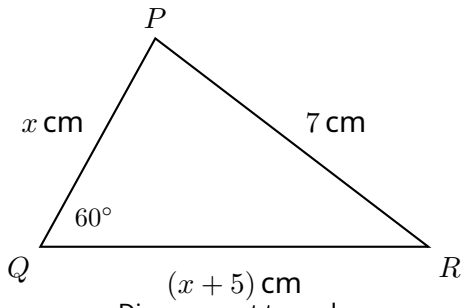


Diagram not to scale

(a) Find the value of x.

$x =$ ..... *(4 marks)*

(b) Hence, find the exact area of triangle PQR. Give your answer in the form $k\sqrt{3}$ cm^2.

.....cm^2 *(2 marks)*

(Total for Question is 6 marks)

Extreme Challenge

In triangle ABC, $AB = x$ cm, $BC = (x + 2)$ cm and angle $ABC = 120°$.
The area of triangle ABC is $6\sqrt{3}$ cm^2.

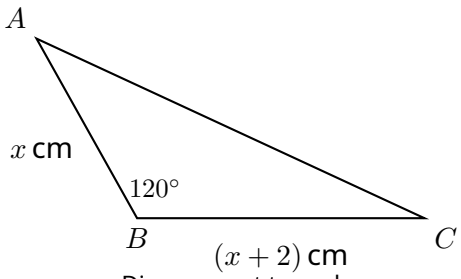


Diagram not to scale

(a) Show that $x^2 + 2x - 24 = 0$, and hence find the value of x.

$x =$ ..... *(3 marks)*

(b) Calculate the exact length of AC. Give your answer in its simplest surd form.

$AC =$ .....cm *(3 marks)*

(Total for Question is 6 marks)

Worked Answers

I Do

1. Apply the cosine rule ($a^2 = b^2 + c^2 - 2bc\cos A$), noting $\cos 60^\circ = \frac{1}{2}$:

$7^2 = x^2 + (x+2)^2 - 2x(x+2)\cos 60^\circ$

$49 = x^2 + (x^2 + 4x + 4) - x(x+2)$

$49 = x^2 + 2x + 4 \implies x^2 + 2x - 45 = 0.$

2. Solve using the quadratic formula (or completing the square):

$(x+1)^2 - 1 - 45 = 0 \implies (x+1)^2 = 46 \implies x = -1 \pm \sqrt{46}.$

Since $x > 0$, $\mathbf{x = \sqrt{46} - 1}$ *(or $-1 + \sqrt{46}$)*.

We Do

$7^2 = x^2 + (x+3)^2 - 2x(x+3)\left(\frac{1}{2}\right)$

$49 = x^2 + x^2 + 6x + 9 - x^2 - 3x \implies x^2 + 3x - 40 = 0 \implies (x+8)(x-5) = 0.$

Since $x > 0$, $\mathbf{x = 5}$.

You Do 1

$8^2 = x^2 + (x+4)^2 - x(x+4)$

$64 = x^2 + 4x + 16 \implies x^2 + 4x - 48 = 0.$

$(x+2)^2 - 4 - 48 = 0 \implies (x+2)^2 = 52 \implies x = -2 \pm \sqrt{52} = -2 \pm 2\sqrt{13}.$

Since $x > 0$, $\mathbf{x = 2\sqrt{13} - 2}$.

You Do 2

Since $\cos 120^\circ = -\frac{1}{2}$, $-2bc\cos 120^\circ = +bc$:

$7^2 = x^2 + (x+2)^2 + x(x+2)$

$49 = x^2 + x^2 + 4x + 4 + x^2 + 2x \implies 3x^2 + 6x - 45 = 0 \implies x^2 + 2x - 15 = 0.$

$(x+5)(x-3) = 0$. Since $x > 0$, $\mathbf{x = 3}$.

You Do 3

$7^2 = x^2 + (2x-1)^2 + x(2x-1)$

$49 = x^2 + 4x^2 - 4x + 1 + 2x^2 - x \implies 7x^2 - 5x - 48 = 0.$

$(7x+16)(x-3) = 0$. Since $x > 0$, $\mathbf{x = 3}$.

Challenge 1

1. Apply the cosine rule with $\cos 60^\circ = \frac{1}{2}$:
$(2x)^2 = (x+1)^2 + (x+3)^2 - (x+1)(x+3)$
$4x^2 = (x^2+2x+1) + (x^2+6x+9) - (x^2+4x+3)$
$4x^2 = x^2 + 4x + 7 \implies 3x^2 - 4x - 7 = 0$. *(Shown)*
2. Factorise: $(3x-7)(x+1) = 0 \implies x = \frac{7}{3}$ or $x = -1$.
Since $x > 0$, $\mathbf{x = \frac{7}{3}}$.

Challenge 2

(a) $7^2 = x^2 + (x+5)^2 - x(x+5)$
$49 = x^2 + 5x + 25 \implies x^2 + 5x - 24 = 0 \implies (x+8)(x-3) = 0$.
Since $x > 0$, $\mathbf{x = 3}$.

(b) Substitute $x = 3$: $PQ = 3$ cm and $QR = 8$ cm.
Area $= \frac{1}{2}ab\sin C = \frac{1}{2}(3)(8)\sin 60^\circ = 12 \times \frac{\sqrt{3}}{2} = \mathbf{6\sqrt{3}}$ cm^2.

Extreme Challenge

(a) Use the area formula Area $= \frac{1}{2}ab\sin C$, noting $\sin 120^\circ = \frac{\sqrt{3}}{2}$:
$\frac{1}{2}x(x+2)\sin 120^\circ = 6\sqrt{3}$
$\frac{1}{2}x(x+2)\left(\frac{\sqrt{3}}{2}\right) = 6\sqrt{3}$
Divide both sides by $\sqrt{3}$ and multiply by 4:
$x(x+2) = 24 \implies x^2 + 2x - 24 = 0$. *(Shown)*
Factorise: $(x+6)(x-4) = 0$. Since $x > 0$, $\mathbf{x = 4}$.

(b) Substitute $x = 4$: $AB = 4$ cm and $BC = 6$ cm.
Use the cosine rule ($\cos 120^\circ = -\frac{1}{2}$):
$AC^2 = 4^2 + 6^2 - 2(4)(6)\left(-\frac{1}{2}\right) = 16 + 36 + 24 = 76$.
$AC = \sqrt{76} = \sqrt{4 \times 19} = \mathbf{2\sqrt{19}}$ cm.