Arithmetic Series

Practice Worksheet (Problem Solving)

I Do

The sum of the first 60 terms of an arithmetic series, S, is 528.

The 55th term of S is 18.6.

The sum of the first X terms of S is 261.

Work out the value of X.
Show your working clearly.

$X =$ .....

(Total for Question is 5 marks)

We Do

The sum of the first 40 terms of an arithmetic series, S, is 230.

The 35th term of S is 13.

The sum of the first X terms of S is 120.

Work out the value of X.
Show your working clearly.

$X =$ .....

(Total for Question is 5 marks)

You Do 1

The sum of the first 20 terms of an arithmetic series, S, is 670.

The 12th term of S is 38.

The sum of the first X terms of S is 390.

Work out the value of X.
Show your working clearly.

$X =$ .....

(Total for Question is 5 marks)

You Do 2

The sum of the first 20 terms of an arithmetic series, S, is 230.

The 10th term of S is 13.

The sum of the first X terms of S is 275.

Work out the value of X.
Show your working clearly.

$X =$ .....

(Total for Question is 5 marks)

You Do 3

The sum of the first 50 terms of an arithmetic series, S, is 485.

The 40th term of S is 18.4.

The sum of the first X terms of S is 111.

Work out the value of X.
Show your working clearly.

$X =$ .....

(Total for Question is 5 marks)

Challenge 1

The first three terms of an arithmetic series are $(2k - 1)$, $(3k + 4)$, and $(6k + 1)$.

(a) Find the value of k.

$k =$ ..... *(2 marks)*

(b) Hence, calculate the sum of the first 25 terms of the series.
Show all of your working clearly.

..... *(3 marks)*

(Total for Question is 5 marks)

Challenge 2

In an arithmetic series, the sum of the first 10 terms is 145.
The sum of the next 10 terms (from the 11th term to the 20th term inclusive) is 445.

(a) Find the first term, a, and the common difference, d, of the series.

$a =$ ..... $d =$ ..... *(3 marks)*

(b) Find the smallest integer value of n for which the sum of the first n terms is greater than 1000.
Show all of your working clearly.

$n =$ ..... *(3 marks)*

(Total for Question is 6 marks)

Extreme Challenge

An arithmetic series has first term a and common difference d.
The 4th term of the series is 43, and the sum of the first 12 terms is 396.

(a) Find the value of a and the value of d.

$a =$ ..... $d =$ ..... *(3 marks)*

(b) Find the **maximum** possible value of S_n, the sum of the first n terms of the series.

..... *(2 marks)*

(c) Find the value of m for which the sum of the first m terms is -90.

$m =$ ..... *(2 marks)*

(Total for Question is 7 marks)

Worked Answers

I Do

1. Form two equations in a and d:
$S_{60} = 30(2a + 59d) = 528 \implies 2a + 59d = 17.6.$
$u_{55} = a + 54d = 18.6 \implies 2a + 108d = 37.2.$
2. Subtract: $49d = 19.6 \implies d = 0.4.$
Substitute into $a + 54(0.4) = 18.6 \implies a + 21.6 = 18.6 \implies a = -3.$
3. Use $S_X = 261$:
$\frac{X}{2}\big(2(-3) + 0.4(X - 1)\big) = 261 \implies X(0.2X - 3.2) = 261.$
Multiply by 5: $X^2 - 16X - 1305 = 0 \implies (X - 45)(X + 29) = 0.$
Since $X > 0$, **X = 45**.

We Do

$S_{40} = 20(2a + 39d) = 230 \implies 2a + 39d = 11.5; \quad u_{35} = a + 34d = 13 \implies 2a + 68d = 26.$
Subtract: $29d = 14.5 \implies d = 0.5$, and $a = 13 - 17 = -4.$
$S_X = \frac{X}{2}\big(-8 + 0.5(X - 1)\big) = 120 \implies X^2 - 17X - 480 = 0 \implies (X - 32)(X + 15) = 0 \implies$ **X = 32**.

You Do 1

$S_{20} = 10(2a + 19d) = 670 \implies 2a + 19d = 67; \quad u_{12} = a + 11d = 38 \implies 2a + 22d = 76.$
Subtract: $3d = 9 \implies d = 3$, and $a = 38 - 33 = 5.$
$S_X = \frac{X}{2}\big(10 + 3(X - 1)\big) = 390 \implies 3X^2 + 7X - 780 = 0 \implies (3X + 52)(X - 15) = 0 \implies$ **X = 15**.

You Do 2

$S_{20} = 10(2a + 19d) = 230 \implies 2a + 19d = 23; \quad u_{10} = a + 9d = 13 \implies 2a + 18d = 26.$
Subtract: $d = -3$, and $a = 13 - 9(-3) = 40.$
$S_X = \frac{X}{2}\big(80 - 3(X - 1)\big) = 275 \implies 3X^2 - 83X + 550 = 0 \implies (3X - 50)(X - 11) = 0.$
Since X is an integer, **X = 11**.

You Do 3

$S_{50} = 25(2a + 49d) = 485 \implies 2a + 49d = 19.4; \quad u_{40} = a + 39d = 18.4 \implies 2a + 78d = 36.8.$
Subtract: $29d = 17.4 \implies d = 0.6$, and $a = 18.4 - 23.4 = -5.$
$S_X = \frac{X}{2}\big(-10 + 0.6(X - 1)\big) = 111 \implies 3X^2 - 53X - 1110 = 0 \implies (3X + 37)(X - 30) = 0 \implies$
X = 30.

Challenge 1

(a) In an arithmetic series, the common difference between consecutive terms is constant:
$(3k+4)-(2k-1)=(6k+1)-(3k+4)$
$k+5=3k-3 \implies 2k=8 \implies$ **k = 4**.

(b) Substitute $k=4$: first three terms are $7, 16, 25$, so $a=7$ and $d=9$.
$S_{25}=\frac{25}{2}\big(2(7)+24(9)\big)=\frac{25}{2}(14+216)=\frac{25}{2}(230)=$ **2875**.

Challenge 2

(a) $S_{10}=5(2a+9d)=145 \implies 2a+9d=29$.
The sum of the first 20 terms is $S_{20}=145+445=590$:
$10(2a+19d)=590 \implies 2a+19d=59$.
Subtract: $10d=30 \implies$ **d = 3**, and $2a+27=29 \implies$ **a = 1**.

(b) $S_n=\frac{n}{2}\big(2+3(n-1)\big)=\frac{n(3n-1)}{2}>1000 \implies 3n^2-n-2000>0$.
Solve $3n^2-n-2000=0$: $n=\frac{1+\sqrt{1+24000}}{6}=\frac{1+154.92\ldots}{6}\approx 25.99$.
Therefore, the smallest integer value is **n = 26** *(check: $S_{25}=925$, $S_{26}=1001$)*.

Extreme Challenge

(a) $u_4=a+3d=43 \implies 2a+6d=86$.
$S_{12}=6(2a+11d)=396 \implies 2a+11d=66$.
Subtract: $5d=-20 \implies$ **d = −4**, and $a+3(-4)=43 \implies$ **a = 55**.

(b) The sum is maximised by adding all positive terms:
$u_n=55-4(n-1)>0 \implies 59-4n>0 \implies n<14.75 \implies n=14$.
$S_{14}=\frac{14}{2}\big(2(55)+13(-4)\big)=7(110-52)=7\times 58=$ **406**.

(c) $S_m=\frac{m}{2}\big(110-4(m-1)\big)=-90 \implies m(57-2m)=-90$.
$2m^2-57m-90=0 \implies (2m+3)(m-30)=0$.
Since $m>0$, **m = 30**.