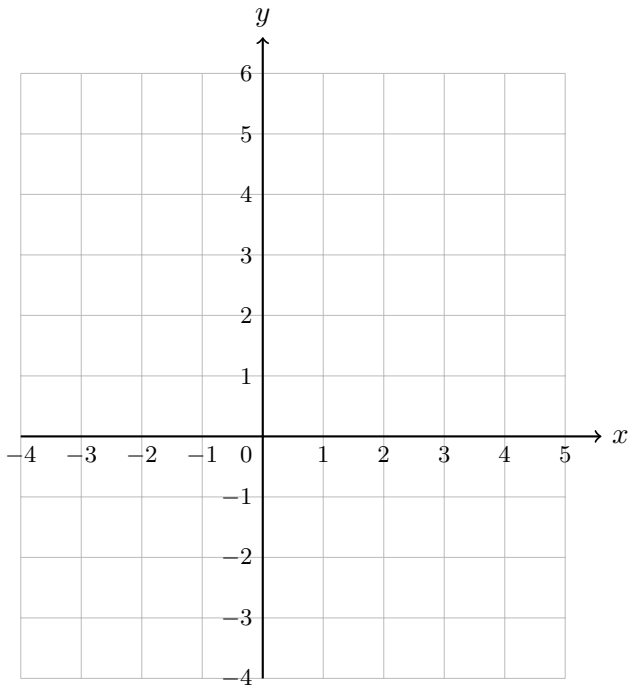
Graphical Inequalities & Regions

Practice Worksheet (Problem Solving)

I Do

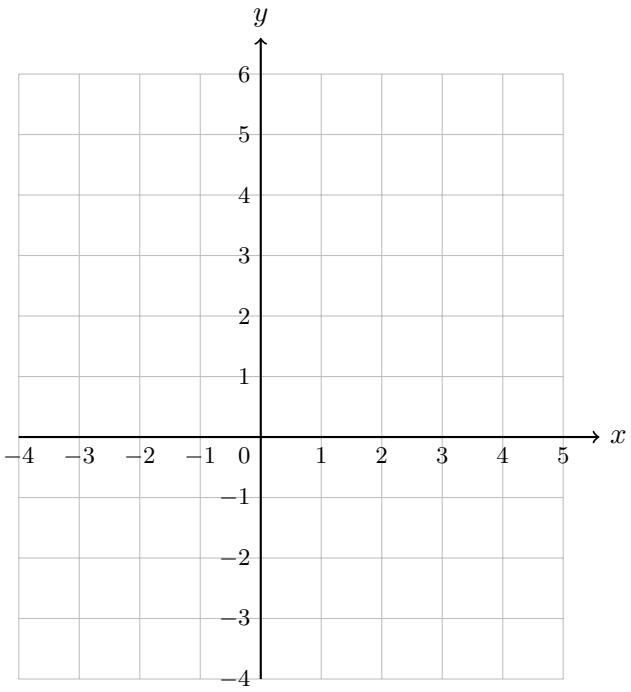
On the grid, show the region that satisfies all three inequalities.
$x \geq 0$
$y \geq x + 1$
$y \leq 4$
Shade the unwanted regions and label the remaining region **R**.



(Total for Question is 3 marks)

We Do

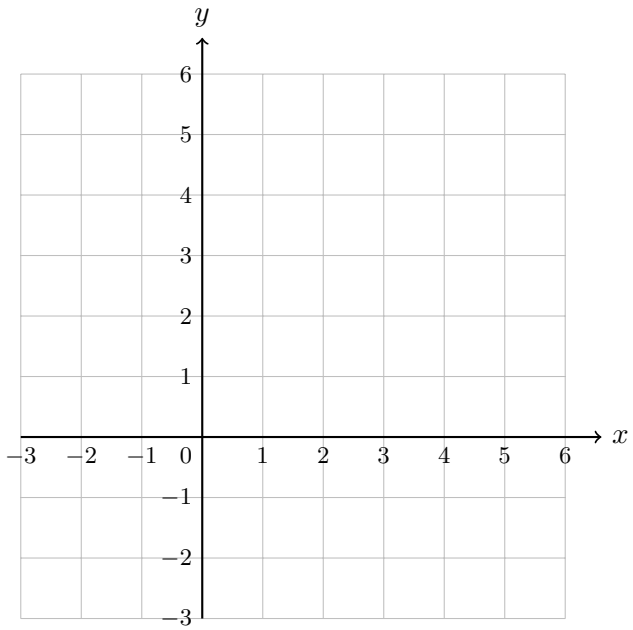
On the grid, show the region that satisfies all three inequalities.
$x \geq -1$
$y \geq x$
$y \leq 3$
Shade the unwanted regions and label the remaining region **R**.



(Total for Question is 3 marks)

You Do 1

On the grid, show the region that satisfies all three inequalities.
$y \geq 0$
$x \leq 4$
$y < x + 1$
Shade the unwanted regions and label the remaining region **R**.



(Total for Question is 3 marks)

You Do 2

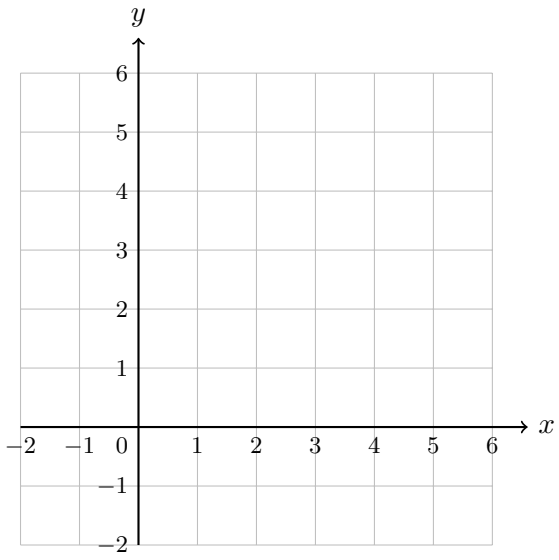
(a) On the grid, show the region **R** that satisfies all three inequalities.

$x > 0$

$y \geq 1$

$x + y \leq 5$

Shade the unwanted regions and label the remaining region **R**.



(3 marks)

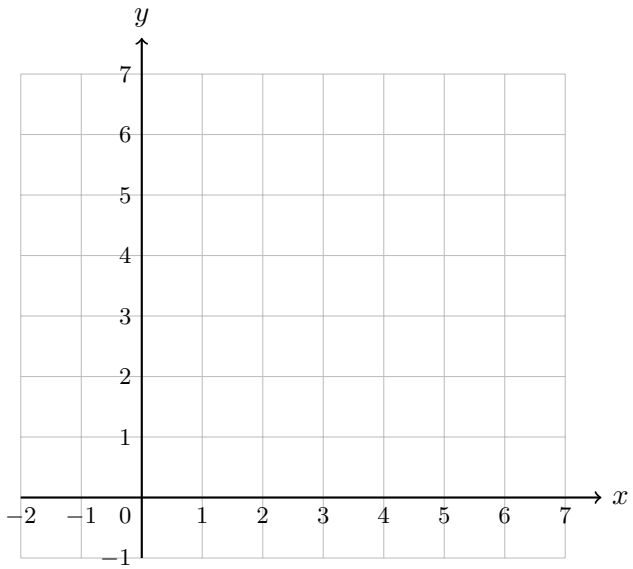
(b) Find the total number of points with integer coordinates that lie in the region **R**.

..... *(1 mark)*

(Total for Question is 4 marks)

You Do 3

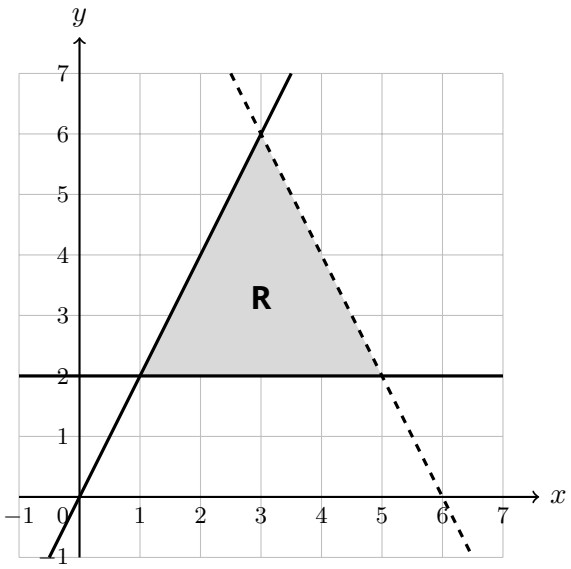
On the grid, show the region that satisfies all three inequalities.
$y \geq 1$
$y \leq 2x + 1$
$y \leq 7 - x$
Shade the unwanted regions and label the remaining region **R**.



(Total for Question is 3 marks)

Challenge 1

The shaded region **R** shown on the grid is defined by three inequalities.



Write down the three inequalities that define the shaded region **R**.

.....

.....

.....

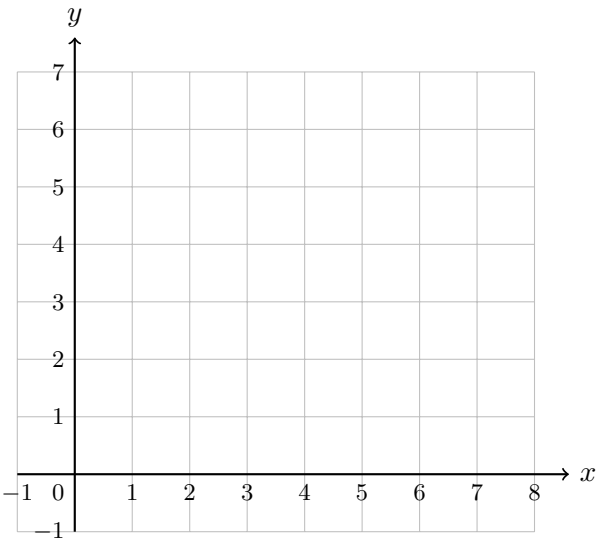
(Total for Question is 4 marks)

Challenge 2

(a) On the grid, show the region **R** that satisfies all four inequalities.

$x \geq 0$ $y \geq 1$ $y \leq 5$ $x + y \leq 7$

Shade the unwanted regions and label the remaining region **R**.



(3 marks)

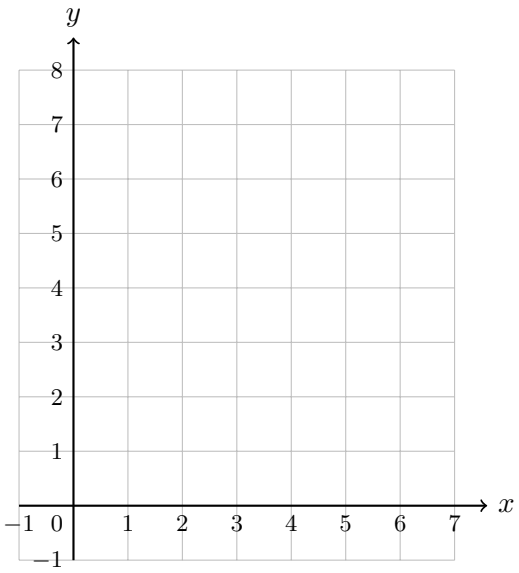
(b) Calculate the exact area of the region **R**.

.....units^2 *(2 marks)*

(Total for Question is 5 marks)

Extreme Challenge

(a) On the grid, show the region **R** that satisfies all four inequalities.
$x \geq 1$ $y \geq 0$ $y \leq x + 2$ $2x + y \leq 11$
Shade the unwanted regions and label the remaining region **R**.



(3 marks)

(b) The point (x, y) lies in the region **R**, where x and y are both integers.
Work out the maximum possible value of $5x + y$, and state the coordinates of the point (x, y) that gives this maximum value.

Maximum value = ... at $(\ldots\ldots, \ldots\ldots)$ *(3 marks)*

(Total for Question is 6 marks)

Worked Answers

I Do
Draw three solid boundary lines:
- $x = 0$ (the y-axis); shade to the left ($x < 0$).
- $y = x + 1$ (passes through $(0, 1)$, $(1, 2)$, $(3, 4)$); shade below the line.
- $y = 4$ (horizontal line through $y = 4$); shade above the line.
Region R is the unshaded triangle with vertices $(\mathbf{0}, \mathbf{1})$, $(\mathbf{0}, \mathbf{4})$, and $(\mathbf{3}, \mathbf{4})$.

We Do
Draw three solid boundary lines:
- $x = -1$ (vertical); shade to the left.
- $y = x$ (passes through $(-1, -1)$, $(0, 0)$, $(3, 3)$); shade below the line.
- $y = 3$ (horizontal); shade above the line.
Region R is the unshaded triangle with vertices $(-\mathbf{1}, -\mathbf{1})$, $(-\mathbf{1}, \mathbf{3})$, and $(\mathbf{3}, \mathbf{3})$.

You Do 1
Draw boundary lines:
- $y = 0$ (solid, along the x-axis); shade below.
- $x = 4$ (solid, vertical); shade to the right.
- $y = x + 1$ (**dashed**, passes through $(-1, 0)$, $(0, 1)$, $(4, 5)$); shade above.
Region R is the unshaded triangle with vertices $(-\mathbf{1}, \mathbf{0})$, $(\mathbf{4}, \mathbf{0})$, and $(\mathbf{4}, \mathbf{5})$ (excluding the slanted edge along $y = x + 1$).

You Do 2
(a) Draw boundary lines:
- $x = 0$ (**dashed**, along the y-axis); shade to the left.
- $y = 1$ (solid); shade below.
- $x + y = 5$ (solid, passes through $(0, 5)$, $(4, 1)$, $(5, 0)$); shade above.
Region R is the unshaded triangle with vertices $(\mathbf{0}, \mathbf{1})$, $(\mathbf{4}, \mathbf{1})$, and $(\mathbf{0}, \mathbf{5})$ (excluding the left edge $x = 0$).
(b) Count all integer coordinate points satisfying the three inequalities ($x = 0$ is excluded since $x > 0$):
- When $x = 1$: $(1, 1), (1, 2), (1, 3), (1, 4)$ (4 points)
- When $x = 2$: $(2, 1), (2, 2), (2, 3)$ (3 points)
- When $x = 3$: $(3, 1), (3, 2)$ (2 points)
- When $x = 4$: $(4, 1)$ (1 point)
Total $= 4 + 3 + 2 + 1 =$ **10 points**.

You Do 3
Draw three solid boundary lines:
- $y = 1$ (horizontal); shade below.
- $y = 2x + 1$ (passes through $(0, 1)$, $(1, 3)$, $(2, 5)$); shade above.
- $y = 7 - x$ (passes through $(2, 5)$, $(6, 1)$, $(7, 0)$); shade above.
Region R is the unshaded triangle with vertices $(\mathbf{0}, \mathbf{1})$, $(\mathbf{6}, \mathbf{1})$, and $(\mathbf{2}, \mathbf{5})$.

Challenge 1

Identify the equation and boundary type of each of the three edges of region **R**:

1. Bottom edge: horizontal solid line $y = 2$. Region **R** is above it $\implies$ **y** $\geq$ **2**.

2. Left edge: solid line passing through $(1, 2)$ and $(3, 6)$.

Gradient $= \dfrac{6-2}{3-1} = 2$, and the line passes through $(0, 0)$, so the equation is $y = 2x$.

Testing $(3, 3)$ inside **R**: $3 \leq 2(3) = 6 \implies$ **y** $\leq$ **2x**.

3. Right edge: **dashed** line passing through $(3, 6)$ and $(5, 2)$.

Gradient $= \dfrac{2-6}{5-3} = -2$. Using $y = -2x + c$ with $(3, 6)$: $6 = -6 + c \implies c = 12$, so line is $y = 12 - 2x$ (or $2x + y = 12$).

Testing $(3, 3)$ inside **R**: $2(3) + 3 = 9 < 12 \implies$ **y** $<$ **12** $-$ **2x** *(or* **2x** $+$ **y** $<$ **12***)*.

Challenge 2

(a) Draw four solid boundary lines:

- $x = 0$ (shade left), $y = 1$ (shade below), $y = 5$ (shade above), and $x + y = 7$ passing through $(2, 5)$ and $(6, 1)$ (shade above).

Region R is the unshaded trapezium with vertices (**0**, **1**), (**0**, **5**), (**2**, **5**), and (**6**, **1**).

(b) The parallel horizontal sides have lengths:

- Top side (along $y = 5$): $2 - 0 = 2$ units.
- Bottom side (along $y = 1$): $6 - 0 = 6$ units.
- Perpendicular height: $5 - 1 = 4$ units.

Area of trapezium $= \dfrac{1}{2}(a + b)h = \dfrac{1}{2}(2 + 6) \times 4 = 4 \times 4 =$ **16 units^2**.

Extreme Challenge

(a) Draw four solid boundary lines:

- $x = 1$ (shade left), $y = 0$ (shade below), $y = x + 2$ passing through $(1, 3)$ and $(3, 5)$ (shade above), and $2x + y = 11$ passing through $(3, 5)$, $(4, 3)$, $(5, 1)$, $(5.5, 0)$ (shade above).

Region R is the unshaded quadrilateral with vertices (**1**, **0**), (**1**, **3**), (**3**, **5**), and (**5**.**5**, **0**).

(b) To maximise $5x + y$, test the integer coordinate points furthest to the right of region **R** (noting that the rightmost vertex $(5.5, 0)$ does **not** have integer coordinates):

- At $(3, 5)$: $5(3) + 5 = 20$.
- At $(4, 3)$: $5(4) + 3 = 23$.
- At $(5, 0)$: $5(5) + 0 = 25$.
- At $(5, 1)$: check boundaries: $2(5) + 1 = 11 \leq 11$ and $1 \leq 5 + 2$, so $(5, 1)$ lies on the boundary of **R**. $5(5) + 1 = 26$.

*(Points with $x \geq 6$ or $(5, 2)$ lie outside **R** since $2x + y > 11$.)*

Answer: Maximum value $=$ **26** at (**5**, **1**).