Algebraic Arithmetic Series

Practice Worksheet (Problem Solving)

I Do

The first term of an arithmetic series is $(4t + 1)$ where $t > 0$.
The nth term of this arithmetic series is $(20t - 7)$.

The common difference of the series is 4.

The sum of the first n terms of the series can be written as $p(qt-1)^r$ where p, q and r are integers.

Find the value of p, the value of q and the value of r.
Show clear algebraic working.

$p =$ ..... $q =$ ..... $r =$ .....

(Total for Question is 5 marks)

We Do

The first term of an arithmetic series is $(6t - 1)$ where $t \geq 1$.
The nth term of this arithmetic series is $(14t - 9)$.

The common difference of the series is 4.

The sum of the first n terms of the series can be written as $p(qt-1)^r$ where p, q and r are integers.

Find the value of p, the value of q and the value of r.
Show clear algebraic working.

$p =$ ..... $q =$ ..... $r =$ .....

(Total for Question is 5 marks)

You Do 1

The first term of an arithmetic series is $(3t + 2)$ where $t > 0$.
The nth term of this arithmetic series is $(9t + 2)$.

The common difference of the series is 2.

The sum of the first n terms of the series can be written as $p(qt+1)^r$ where p, q and r are integers.

Find the value of p, the value of q and the value of r.
Show clear algebraic working.

$p =$ ..... $q =$ ..... $r =$ .....

(Total for Question is 5 marks)

You Do 2

The first term of an arithmetic series is $(3t + 1)$ where $t \geq 1$.
The nth term of this arithmetic series is $(13t - 9)$.

The common difference of the series is 5.

The sum of the first n terms of the series can be written as $p(qt-1)^r$ where p, q and r are integers.

Find the value of p, the value of q and the value of r.
Show clear algebraic working.

$p =$ ..... $q =$ ..... $r =$ .....

(Total for Question is 5 marks)

You Do 3

The first term of an arithmetic series is $(3k + 2)$ where $k \geq 1$.
The second term of this arithmetic series is $(3k + 8)$.
The nth term of this arithmetic series is $(21k - 10)$.

The sum of the first n terms of the series can be written as $p(qk-1)^r$ where p, q and r are integers.

Find the value of p, the value of q and the value of r.
Show clear algebraic working.

$p =$ ..... $q =$ ..... $r =$ .....

(Total for Question is 5 marks)

Challenge 1

The first term of an arithmetic series is $(21t - 9)$ where $t \geq 1$.
The common difference of the series is -4.
The nth term of this arithmetic series is $(9t - 1)$.

(a) Find an expression for n in terms of t.

$n =$ ..... *(2 marks)*

(b) Show that the sum of the first n terms of the series can be written in the form $p(qt - 1)^r$, stating the integer values of p, q and r.

$p =$ ..... $q =$ ..... $r =$ ..... *(3 marks)*

(Total for Question is 5 marks)

Challenge 2

An arithmetic series has first term t and common difference $2t$, where t is a positive integer.
The nth term of this series is $(8t^2 - t)$.

(a) Show that the sum of the first n terms of the series can be written in the form pt^r, where p and r are integers to be found.
Show clear algebraic working.

$p =$ ..... $r =$ ..... *(4 marks)*

(b) Given that the sum of the first n terms of this series is 432, find the value of t.

$t =$ ..... *(2 marks)*

(Total for Question is 6 marks)

Extreme Challenge

The first term of an arithmetic series is $(5t + 1)$ where t is a positive integer.
The common difference of the series is 4.
The nth term of this arithmetic series is $(25t - 7)$.

(a) Express the sum of the first n terms of the series in the form $p(qt - 1)^r$, where p, q and r are integers.

..... *(3 marks)*

(b) Given that the sum of the first n terms is $1\,083$, work out the value of t and hence find the value of n.

$t =$ ..... $n =$ ..... *(2 marks)*

(c) Find the value of the middle term of this series.

..... *(2 marks)*

(Total for Question is 7 marks)

Worked Answers

I Do

1. Use $u_n = a + (n-1)d$ to find n in terms of t:
$20t - 7 = (4t + 1) + 4(n - 1) \implies 16t - 8 = 4(n - 1) \implies n - 1 = 4t - 2 \implies n = 4t - 1.$
2. Use $S_n = \dfrac{n}{2}(a + u_n)$:
$a + u_n = (4t + 1) + (20t - 7) = 24t - 6 = 6(4t - 1).$
$S_n = \dfrac{4t - 1}{2} \times 6(4t - 1) = 3(4t - 1)^2.$
Answer: p $= 3$, **q** $= 4$, **r** $= 2$.

We Do

1. $14t - 9 = (6t - 1) + 4(n - 1) \implies 8t - 8 = 4(n - 1) \implies n - 1 = 2t - 2 \implies n = 2t - 1.$
2. $a + u_n = (6t - 1) + (14t - 9) = 20t - 10 = 10(2t - 1).$
$S_n = \dfrac{2t - 1}{2} \times 10(2t - 1) = 5(2t - 1)^2.$
Answer: p $= 5$, **q** $= 2$, **r** $= 2$.

You Do 1

1. $9t + 2 = (3t + 2) + 2(n - 1) \implies 6t = 2(n - 1) \implies n - 1 = 3t \implies n = 3t + 1.$
2. $a + u_n = (3t + 2) + (9t + 2) = 12t + 4 = 4(3t + 1).$
$S_n = \dfrac{3t + 1}{2} \times 4(3t + 1) = 2(3t + 1)^2.$
Answer: p $= 2$, **q** $= 3$, **r** $= 2$.

You Do 2

1. $13t - 9 = (3t + 1) + 5(n - 1) \implies 10t - 10 = 5(n - 1) \implies n - 1 = 2t - 2 \implies n = 2t - 1.$
2. $a + u_n = (3t + 1) + (13t - 9) = 16t - 8 = 8(2t - 1).$
$S_n = \dfrac{2t - 1}{2} \times 8(2t - 1) = 4(2t - 1)^2.$
Answer: p $= 4$, **q** $= 2$, **r** $= 2$.

You Do 3

1. Find the common difference: $d = (3k + 8) - (3k + 2) = 6.$
2. $21k - 10 = (3k + 2) + 6(n - 1) \implies 18k - 12 = 6(n - 1) \implies n - 1 = 3k - 2 \implies n = 3k - 1.$
3. $a + u_n = (3k + 2) + (21k - 10) = 24k - 8 = 8(3k - 1).$
$S_n = \dfrac{3k - 1}{2} \times 8(3k - 1) = 4(3k - 1)^2.$
Answer: p $= 4$, **q** $= 3$, **r** $= 2$.

Challenge 1

(a) Substitute $a = 21t - 9$, $u_n = 9t - 1$, and $d = -4$ into $u_n = a + (n-1)d$:
$9t - 1 = (21t - 9) - 4(n-1)$
$-12t + 8 = -4(n-1)$
Divide both sides by -4: $n - 1 = 3t - 2 \implies \mathbf{n = 3t - 1}$.

(b) Use $S_n = \dfrac{n}{2}(a + u_n)$:
$a + u_n = (21t - 9) + (9t - 1) = 30t - 10 = 10(3t - 1)$.
$S_n = \dfrac{3t-1}{2} \times 10(3t - 1) = 5(3t - 1)^2$.
Answer: p = 5, q = 3, r = 2.

Challenge 2

(a) 1. Find n in terms of t using $u_n = a + (n-1)d$:
$8t^2 - t = t + 2t(n-1) \implies 8t^2 - 2t = 2t(n-1)$.
Since $t > 0$, divide both sides by $2t$: $4t - 1 = n - 1 \implies n = 4t$.
2. Substitute into $S_n = \dfrac{n}{2}(a + u_n)$:
$a + u_n = t + (8t^2 - t) = 8t^2$.
$S_n = \dfrac{4t}{2}(8t^2) = 2t(8t^2) = 16t^3$.
Answer: p = 16, r = 3.

(b) Set $16t^3 = 432$:
$t^3 = \dfrac{432}{16} = 27 \implies \mathbf{t = 3}$.

Extreme Challenge

(a) 1. Find n: $25t - 7 = (5t+1) + 4(n-1) \implies 20t - 8 = 4(n-1) \implies n - 1 = 5t - 2 \implies n = 5t - 1$.
2. $a + u_n = (5t + 1) + (25t - 7) = 30t - 6 = 6(5t - 1)$.
3. $S_n = \dfrac{5t-1}{2} \times 6(5t - 1) = \mathbf{3(5t - 1)^2}$ $\quad (p = 3,\ q = 5,\ r = 2)$.

(b) Set $3(5t - 1)^2 = 1\,083$:
$(5t - 1)^2 = 361 \implies 5t - 1 = \pm 19$.
Since $t > 0$, $5t - 1 = 19 \implies 5t = 20 \implies \mathbf{t = 4}$.
Number of terms: $n = 5(4) - 1 = \mathbf{19}$.

(c) With $n = 19$, the middle term is the $\dfrac{19 + 1}{2} = 10$th term.
Since $t = 4$, the first term is $a = 5(4) + 1 = 21$ and $d = 4$:
$u_{10} = 21 + 9(4) = 21 + 36 = \mathbf{57}$.
(Alternatively, the middle term equals the mean of all 19 terms: $1\,083 \div 19 = 57$.)