Percentage Change in Formulas

Practice Worksheet (Edexcel 4MA1 Style)

I Do

Keeley is asked to solve the following problem.

The variable P is related to variables q and r by the formula

$$P = \frac{kq^3}{\sqrt{r}}$$

where k is a constant.

q is increased by 20%.
r is decreased by 36%.

Work out the percentage increase in the value of P.

.....%

(Total for Question is 4 marks)

We Do

Catherine is given the formula

$$V = \frac{ka^2}{\sqrt{b}}$$

where k is a constant.

a is increased by 20%.
b is decreased by 64%.

Work out the percentage increase in the value of V.

.....%

(Total for Question is 4 marks)

You Do 1

John uses the mathematical relation

$$T = kx\sqrt{y}$$

where k is a constant.

x is increased by 25%.
y is increased by 44%.

Work out the percentage increase in the value of T.

.....%

(Total for Question is 3 marks)

You Do 2

Ryan is working with the formula

$$W = \frac{ku^3}{v}$$

where k is a constant.

u is increased by 10%.
v is increased by 21%.

Work out the percentage change in the value of W.
You must state whether it is an increase or a decrease.

.....%

(Total for Question is 3 marks)

You Do 3

Paul knows that the kinetic energy E of an object is given by the formula

$$E = \frac{1}{2}mv^2$$

where m is the mass and v is the velocity.

The mass m of the object is decreased by 20%.
The velocity v of the object is increased by 50%.

Work out the percentage increase in the kinetic energy E.

.....%

(Total for Question is 3 marks)

Challenge 1

Sophie investigates a physical force F given by the formula

$$F = \frac{kA}{d^2}$$

where k is a constant.

The value of A is increased by 25%.
The value of d is changed such that the overall force F **decreases** by 20%.

Work out the percentage change in the value of d.

.....%

(Total for Question is 4 marks)

Challenge 2

Paige studies an electrical resistance formula

$$R = \frac{kL^2}{\sqrt{A}}$$

where k is a constant.

The length L is **decreased** by 20%.
The resistance R **increases** by 60%.

Work out the percentage decrease in the cross-sectional area A.
You must show all your working clearly.

.....%

(Total for Question is 4 marks)

Extreme Challenge

Katie analyses the mathematical relation

$$M = \frac{kx^2}{\sqrt{y}}$$

where k is a constant.

The value of x is increased by 20%.
The overall value of M is increased by 125%.

Work out the exact percentage decrease in the value of y.
You must show all your algebraic working clearly.

.....%

(Total for Question is 5 marks)

Worked Answers

I Do

Let the original multipliers for q and r be 1.
q is increased by 20% $\implies$ new q multiplier is 1.2.
r is decreased by 36% $\implies$ new r multiplier is $1 - 0.36 = 0.64$.
Substitute these multipliers into the variable parts of the formula:
Multiplier for $P = \frac{(1.2)^3}{\sqrt{0.64}} = \frac{1.728}{0.8}$.
To divide 1.728 by 0.8, multiply top and bottom by 10: $\frac{17.28}{8} = 2.16$.
A multiplier of 2.16 represents an increase of $(2.16 - 1) \times 100 = \mathbf{116}\%$.

We Do

a is increased by 20% $\implies$ new a multiplier is 1.2.
b is decreased by 64% $\implies$ new b multiplier is $1 - 0.64 = 0.36$.
Substitute into the formula structure for V:
Multiplier for $V = \frac{(1.2)^2}{\sqrt{0.36}} = \frac{1.44}{0.6}$.
$\frac{1.44}{0.6} = \frac{14.4}{6} = 2.4$.
A multiplier of 2.4 represents an increase of $(2.4 - 1) \times 100 = \mathbf{140}\%$.

You Do 1

x is increased by 25% $\implies$ multiplier is 1.25.
y is increased by 44% $\implies$ multiplier is 1.44.
Substitute into the formula structure for T:
Multiplier for $T = 1.25 \times \sqrt{1.44} = 1.25 \times 1.2$.
$1.25 \times 1.2 = 1.50$.
A multiplier of 1.5 means a percentage increase of $\mathbf{50}\%$.

You Do 2

u is increased by 10% $\implies$ multiplier is 1.1.
v is increased by 21% $\implies$ multiplier is 1.21.
Substitute into the formula structure for W:
Multiplier for $W = \frac{(1.1)^3}{1.21} = \frac{1.331}{1.21}$.
$\frac{1.331}{1.21} = 1.1$.
A multiplier of 1.1 represents an **increase of 10%**.

You Do 3

m is decreased by 20% $\implies$ multiplier is 0.8.
v is increased by 50% $\implies$ multiplier is 1.5.
Substitute into the formula structure for E (the $\frac{1}{2}$ is a constant so it is ignored for percentage change multipliers):
Multiplier for $E = 0.8 \times (1.5)^2 = 0.8 \times 2.25$.
$0.8 \times 2.25 = 1.8$.
A multiplier of 1.8 represents an increase of **80**%.

Challenge 1

A multiplier $= 1.25$.
Let the multiplier for d be x.
Overall F multiplier $= 0.8$ (since it decreases by 20%).
Set up the equation using the formula structure:
$0.8 = \frac{1.25}{x^2}$
$x^2 = \frac{1.25}{0.8} = \frac{12.5}{8} = \frac{125}{80} = \frac{25}{16}$.
$x = \sqrt{\frac{25}{16}} = \frac{5}{4} = 1.25$.
A multiplier of 1.25 for d means that d **increases by 25%**.

Challenge 2

L multiplier $= 0.8$ (decreased by 20%).
Let the multiplier for A be y.
Overall R multiplier $= 1.6$ (increased by 60%).
Set up the equation using the formula structure:
$1.6 = \frac{(0.8)^2}{\sqrt{y}}$
$1.6 = \frac{0.64}{\sqrt{y}}$
$\sqrt{y} = \frac{0.64}{1.6} = \frac{6.4}{16} = 0.4$.
Square both sides:
$y = (0.4)^2 = 0.16$.
A multiplier of 0.16 represents a decrease of $1 - 0.16 = 0.84 =$ **84**%.

Extreme Challenge

x multiplier $= 1.2$.
Let the multiplier for y be w.
Overall M multiplier $= 2.25$ (increased by 125%).
Set up the equation:
$2.25 = \frac{(1.2)^2}{\sqrt{w}}$
$2.25 = \frac{1.44}{\sqrt{w}}$
$\sqrt{w} = \frac{1.44}{2.25} = \frac{144}{225}$.
Simplify the fraction (divide by 9): $\frac{16}{25} = 0.64$.
$\sqrt{w} = 0.64$.
Square both sides to find w:
$w = 0.64^2 = 0.4096$.
A multiplier of 0.4096 represents a decrease of $1 - 0.4096 = 0.5904$.
Therefore, y decreases by **59.04**%.