Similar Shapes: Cones and Frustums

Practice Worksheet (Edexcel 4MA1 Style)

I Do

John has a solid cone with a base radius of R cm and a vertical height of H cm.

A smaller cone of vertical height h cm is removed from the top of the solid cone to leave a frustum.

The volume of the frustum is $\frac{19}{27}$ of the volume of the original large cone.

The total surface area of the original large cone is 540π cm^2.

Work out the total surface area of the smaller cone that was removed.
Give your answer in terms of π.

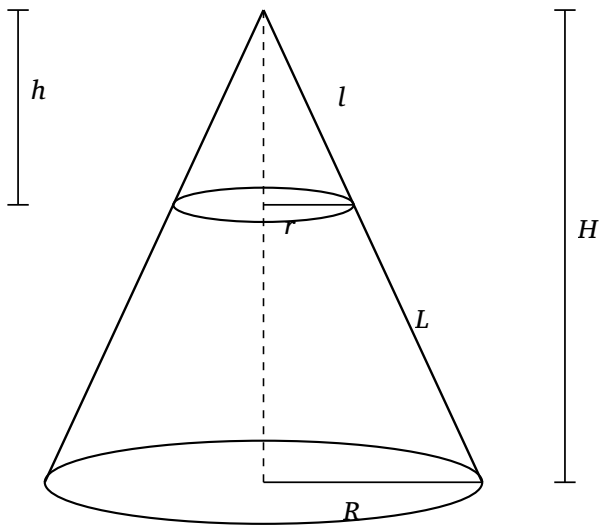


Diagram not accurately drawn

Formulae for any cone

Volume

$$V = \frac{1}{3}\pi r^2 h$$

Curved surface area

$$\text{CSA} = \pi r l$$

Total surface area

$$\text{TSA} = \pi r^2 + \pi r l$$

r: radius; h: vertical height;
l: slant height of the cone.

..... cm^2

(Total for Question is 4 marks)

We Do

Catherine has a solid cone with a base radius of R cm and a vertical height of H cm.

A smaller cone of vertical height h cm is removed from the top of the solid cone to leave a frustum.

The volume of the frustum is $\frac{63}{64}$ of the volume of the original large cone.

The total surface area of the original large cone is 320π cm^2.

Work out the total surface area of the smaller cone that was removed.
Give your answer in terms of π.

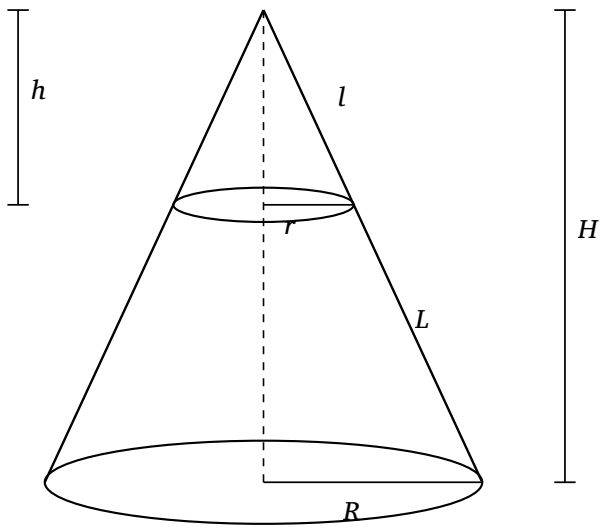


Diagram not accurately drawn

Formulae for any cone

Volume

$$V = \frac{1}{3}\pi r^2 h$$

Curved surface area

$$\text{CSA} = \pi r l$$

Total surface area

$$\text{TSA} = \pi r^2 + \pi r l$$

r: radius; h: vertical height;
l: slant height of the cone.

..... cm^2

(Total for Question is 4 marks)

You Do 1

John has a solid cone. A smaller cone is removed from the top to leave a frustum.

The volume of the frustum is $\frac{7}{8}$ of the volume of the original large cone.

The total surface area of the original large cone is $160\pi\,\text{cm}^2$.

Work out the total surface area of the smaller cone that was removed.
Give your answer in terms of π.

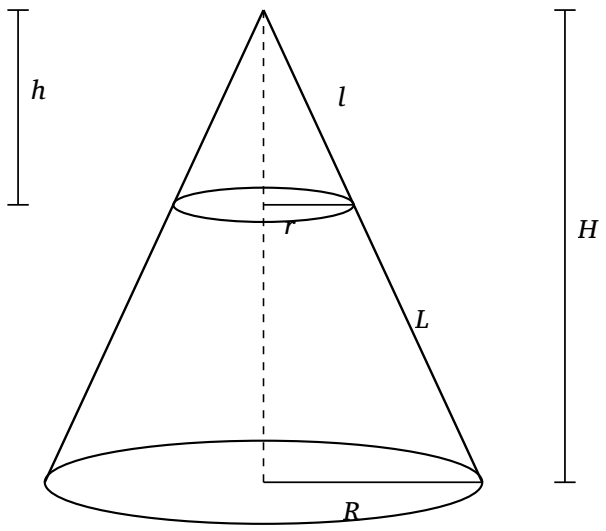


Diagram not accurately drawn

Formulae for any cone

Volume

$$V = \frac{1}{3}\pi r^2 h$$

Curved surface area

$$\text{CSA} = \pi r l$$

Total surface area

$$\text{TSA} = \pi r^2 + \pi r l$$

r: radius; h: vertical height;
l: slant height of the cone.

..... cm^2

(Total for Question is 3 marks)

You Do 2

Ryan starts with a solid large cone and removes a smaller cone from the top to create a frustum.

The curved surface area of the smaller cone is $\frac{16}{49}$ of the curved surface area of the original large cone.

The volume of the original large cone is $686\pi\,\text{cm}^3$.

Work out the volume of the **frustum**.
Give your answer in terms of π.

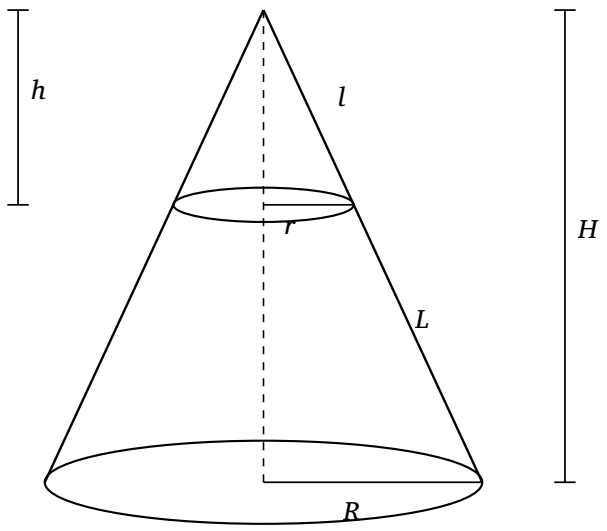


Diagram not accurately drawn

Formulae for any cone

Volume

$$V = \frac{1}{3}\pi r^2 h$$

Curved surface area

$$\text{CSA} = \pi r l$$

Total surface area

$$\text{TSA} = \pi r^2 + \pi r l$$

r: radius; h: vertical height;
l: slant height of the cone.

..... cm^3

(Total for Question is 4 marks)

You Do 3

Paul has a solid large cone. He removes a smaller cone from the top to leave a frustum.

The volume of the frustum is $\frac{61}{125}$ of the volume of the original large cone.

The total surface area of the **smaller** cone is $320\pi\,\text{cm}^2$.

Work out the total surface area of the **original large** cone.
Give your answer in terms of π.

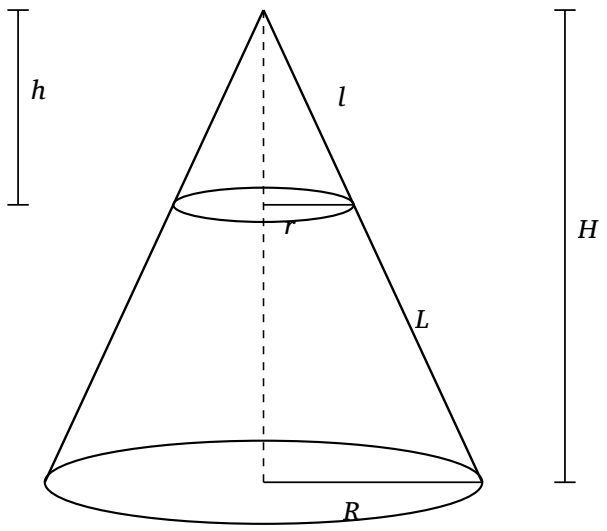


Diagram not accurately drawn

Formulae for any cone

Volume

$$V = \frac{1}{3}\pi r^2 h$$

Curved surface area

$$\text{CSA} = \pi r l$$

Total surface area

$$\text{TSA} = \pi r^2 + \pi r l$$

r: radius; h: vertical height;
l: slant height of the cone.

.....cm^2

(Total for Question is 4 marks)

Challenge 1

Sophie has a solid cone. She cuts off a smaller cone from the top to create a frustum.

The volume of the frustum is $\frac{26}{27}$ of the volume of the original large cone.

Find the ratio of the Curved Surface Area of the **frustum** to the Curved Surface Area of the **original large cone**.
Give your ratio in its simplest integer form.

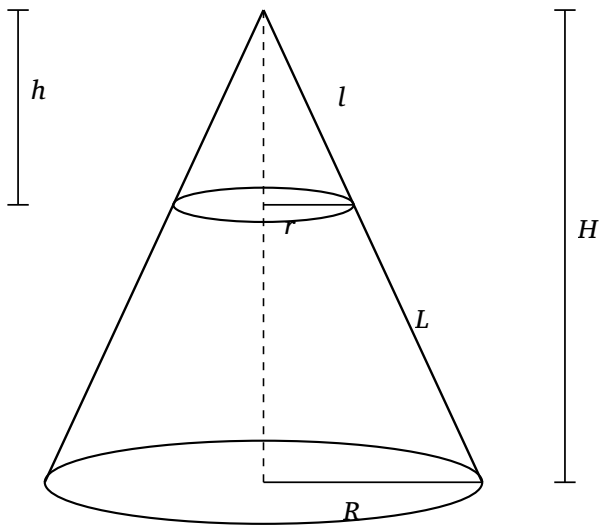


Diagram not accurately drawn

Formulae for any cone

Volume

$$V = \frac{1}{3}\pi r^2 h$$

Curved surface area

$$\text{CSA} = \pi r l$$

Total surface area

$$\text{TSA} = \pi r^2 + \pi r l$$

r: radius; h: vertical height;
l: slant height of the cone.

.....: .....

(Total for Question is 4 marks)

Challenge 2

Paige makes a solid frustum by removing a small cone from the top of a large cone.

The radius of the top circular face of the frustum is 3 cm.
The radius of the bottom circular face of the frustum is 6 cm.

The volume of the frustum is 210π cm^3.

Calculate the vertical height of the frustum.
You must show all your algebraic working.

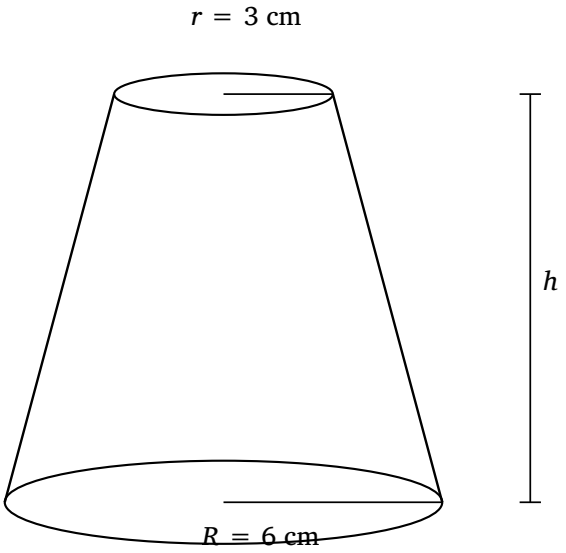


Diagram not accurately drawn

Formulae for any cone

Volume

$$V = \frac{1}{3}\pi r^2 h$$

Curved surface area

$$\text{CSA} = \pi r l$$

Total surface area

$$\text{TSA} = \pi r^2 + \pi r l$$

Frustum volume:
$V = \frac{1}{3}\pi h(R^2 + Rr + r^2)$

..... cm

(Total for Question is 5 marks)

Extreme Challenge

Katie has a solid cone of vertical height H.

She makes two horizontal cuts parallel to the base, dividing the vertical height H into three equal parts.

This cuts the original cone into three solid pieces:

- A small cone at the top.
- A middle frustum.
- A bottom frustum.

Work out the ratio of:
(Volume of the small cone) : (Volume of the middle frustum) : (Volume of the bottom frustum)

Give your ratio in its simplest form.

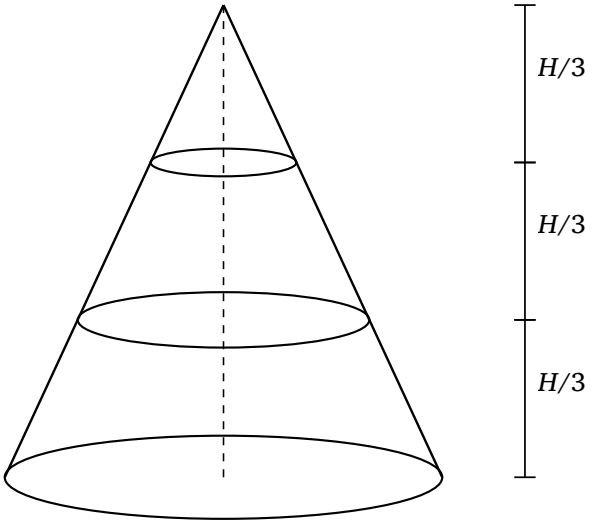


Diagram not accurately drawn

Formulae for any cone

Volume

$$V = \frac{1}{3}\pi r^2 h$$

Curved surface area

$$\text{CSA} = \pi r l$$

Total surface area

$$\text{TSA} = \pi r^2 + \pi r l$$

r: radius; h: vertical height;
l: slant height of the cone.

.....: .....: .....

(Total for Question is 5 marks)

Worked Answers

I Do

The volume of the large cone is 1 (or $\frac{27}{27}$).
Volume of small cone = Volume of large cone − Volume of frustum
Volume of small cone $= 1 - \frac{19}{27} = \frac{8}{27}$ of the large cone.
Volume Scale Factor (VSF) $= \frac{8}{27}$.
Linear Scale Factor (LSF) $= \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$.
Area Scale Factor (ASF) $= \left(\frac{2}{3}\right)^2 = \frac{4}{9}$.
Total surface area of small cone $= \frac{4}{9} \times 540\pi = 4 \times 60\pi = \mathbf{240\pi\ cm^2}$.

We Do

Volume of small cone $= 1 - \frac{63}{64} = \frac{1}{64}$ of the large cone.
Volume Scale Factor (VSF) $= \frac{1}{64}$.
Linear Scale Factor (LSF) $= \sqrt[3]{\frac{1}{64}} = \frac{1}{4}$.
Area Scale Factor (ASF) $= \left(\frac{1}{4}\right)^2 = \frac{1}{16}$.
Total surface area of small cone $= \frac{1}{16} \times 320\pi = \mathbf{20\pi\ cm^2}$.

You Do 1

Volume of small cone $= 1 - \frac{7}{8} = \frac{1}{8}$ of the large cone.
Volume Scale Factor (VSF) $= \frac{1}{8}$.
Linear Scale Factor (LSF) $= \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$.
Area Scale Factor (ASF) $= \left(\frac{1}{2}\right)^2 = \frac{1}{4}$.
Total surface area of small cone $= \frac{1}{4} \times 160\pi = \mathbf{40\pi\ cm^2}$.

You Do 2

Area Scale Factor (ASF) $= \frac{16}{49}$.
Linear Scale Factor (LSF) $= \sqrt{\frac{16}{49}} = \frac{4}{7}$.
Volume Scale Factor (VSF) $= \left(\frac{4}{7}\right)^3 = \frac{64}{343}$.
Volume of small cone $= \frac{64}{343} \times 686\pi = 64 \times 2\pi = 128\pi\,\text{cm}^3$.
Volume of frustum = Volume of large − Volume of small $= 686\pi - 128\pi = \mathbf{558\pi\ cm^3}$.

You Do 3

Volume of small cone $= 1 - \frac{61}{125} = \frac{64}{125}$ of the large cone.
Volume Scale Factor (VSF) for small to large $= \frac{64}{125}$.
Linear Scale Factor (LSF) $= \sqrt[3]{\frac{64}{125}} = \frac{4}{5}$.
Area Scale Factor (ASF) $= \left(\frac{4}{5}\right)^2 = \frac{16}{25}$.
This means: Area of small $= \frac{16}{25}\times$ Area of large.
$320\pi = \frac{16}{25} \times$ Area of large.
Area of large $= 320\pi \times \frac{25}{16} = 20\pi \times 25 = \mathbf{500\pi\ cm^2}$.

Challenge 1

Volume of small cone $= 1 - \frac{26}{27} = \frac{1}{27}$ of the large cone.
VSF $= \frac{1}{27} \implies$ LSF $= \frac{1}{3} \implies$ ASF $= \frac{1}{9}$.
Curved Surface Area (CSA) of small cone $= \frac{1}{9}$ of CSA of large cone.
CSA of frustum = CSA of large cone − CSA of small cone
CSA of frustum $= 1 - \frac{1}{9} = \frac{8}{9}$ of CSA of large cone.
Ratio of CSA frustum : CSA large cone $= \frac{8}{9} : 1 = \mathbf{8} : \mathbf{9}$.

Challenge 2

The top radius is 3 cm and bottom is 6 cm.
Linear Scale Factor (small cone to large cone) $= \frac{3}{6} = \frac{1}{2}$.
Volume Scale Factor $= \left(\frac{1}{2}\right)^3 = \frac{1}{8}$.
Let the volume of the large cone be V. Volume of small cone $= \frac{1}{8}V$.
Volume of frustum $= V - \frac{1}{8}V = \frac{7}{8}V$.
We are given the volume of the frustum is 210π:
$\frac{7}{8}V = 210\pi \implies V = \frac{210\pi \times 8}{7} = 240\pi\,\text{cm}^3$.
For the large cone, $V = \frac{1}{3}\pi r^2 H \implies 240\pi = \frac{1}{3}\pi(6)^2 H$.
$240\pi = 12\pi H \implies H = 20\,\text{cm}$.
Since LSF is $\frac{1}{2}$, the small cone has height 10 cm.
Height of frustum $= 20 - 10 = \mathbf{10\ cm}$.

Extreme Challenge

Let the height of the three sections be h, so the total height is $3h$.
There are three similar cones from the top down:

- Cone 1 (top): height h, Volume $V_1 \propto 1^3 = 1$
- Cone 2 (top + middle): height $2h$, Volume $V_2 \propto 2^3 = 8$
- Cone 3 (whole thing): height $3h$, Volume $V_3 \propto 3^3 = 27$

Volume of small cone $= V_1 = 1$
Volume of middle frustum $= V_2 - V_1 = 8 - 1 = 7$
Volume of bottom frustum $= V_3 - V_2 = 27 - 8 = 19$
Ratio $= \mathbf{1} : \mathbf{7} : \mathbf{19}$.