Permutations with Restrictions

Practice Worksheet (Edexcel 1MA1 Style)

I Do

A 4-digit security code is to be formed using digits chosen from the set

$$\{1, 2, 3, 4, 5, 6, 7, 8\}$$

No digit may be used more than once in any code.
The 4-digit code must be an even number greater than 5000.

Work out the total number of possible codes that can be formed.

.....

(Total for Question is 4 marks)

We Do

A 4-digit security code is to be formed using digits chosen from the set

$$\{1, 2, 3, 4, 5, 6, 7\}$$

No digit may be used more than once in any code.
The 4-digit code must be an odd number greater than 4000.

Work out the total number of possible codes that can be formed.

.....

(Total for Question is 4 marks)

You Do 1

Tom is forming a 3-digit security code using digits chosen from the set

$$\{1, 2, 3, 4, 5\}$$

No digit may be used more than once in any code.
The 3-digit code must be an even number greater than 300.

Work out the total number of possible codes that Tom can form.

.....

(Total for Question is 3 marks)

You Do 2

Zoe wants to create a 4-digit passcode using digits chosen from the set

$$\{2, 3, 4, 5, 6, 7, 8, 9\}$$

No digit may be used more than once in her passcode.
The 4-digit passcode must be an even number less than 5000.

Work out the total number of possible passcodes Zoe could create.

. . . . . .

(Total for Question is 4 marks)

You Do 3

Mark is writing a 5-digit ID number using digits chosen from the set

$$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

No digit may be used more than once in the ID number.
The 5-digit ID number must be an odd number greater than 70000.

Work out the total number of possible ID numbers Mark can form.

.....

(Total for Question is 4 marks)

Challenge 1

Emma forms a 3-digit number using digits chosen from the set

$$\{1, 2, 3, 4, 5, 6, 7\}$$

No digit may be used more than once.
She wants the number to be a multiple of 5, AND it must be less than 600.

Work out the total number of possible 3-digit numbers Emma can form.

.....

(Total for Question is 3 marks)

Challenge 2

Ben wants to form a 4-digit number using digits from the set

$$\{1, 2, 3, 4, 5, 6\}$$

No digit may be used more than once.
The number must be greater than 4000, and it must be a multiple of 4.
(Hint: A number is a multiple of 4 if its last two digits form a multiple of 4).

Work out the total number of possible 4-digit numbers Ben can form.
You must show all your working clearly.

. . . . . .

(Total for Question is 5 marks)

Extreme Challenge

Keeley forms a 6-digit number by using all the digits from the set

$$\{1, 2, 3, 4, 5, 6\}$$

exactly once each.

Find the number of possible 6-digit numbers she can form which are odd AND greater than 300,000.

You must show all your working clearly.

.....

(Total for Question is 5 marks)

Worked Answers

I Do

The code is 4 digits. Available: $\{1, 2, 3, 4, 5, 6, 7, 8\}$ (8 digits).
To be > 5000, the first digit must be 5, 6, 7, or 8.
To be even, the last digit must be 2, 4, 6, or 8.
Because 6 and 8 appear in both condition sets, we split into two cases based on the first digit.

Case 1: First digit is odd (5 or 7)

- Choice for 1st digit = 2 choices (5, 7).
- Choice for 4th digit = 4 choices (2, 4, 6, 8 are all still available).
- Choice for 2nd digit = 6 choices remaining.
- Choice for 3rd digit = 5 choices remaining.
- Number of codes $= 2 \times 6 \times 5 \times 4 = 240$.

Case 2: First digit is even (6 or 8)

- Choice for 1st digit = 2 choices (6, 8).
- Choice for 4th digit = 3 choices (one even digit was used up at the front).
- Choice for 2nd digit = 6 choices remaining.
- Choice for 3rd digit = 5 choices remaining.
- Number of codes $= 2 \times 6 \times 5 \times 3 = 180$.

Total possible codes $= 240 + 180 =$ **420**.

We Do

Available: $\{1, 2, 3, 4, 5, 6, 7\}$ (7 digits).
To be > 4000, first digit is 4, 5, 6, 7. To be odd, last digit is 1, 3, 5, 7.
Split into cases because 5 and 7 overlap.

Case 1: First digit is even (4 or 6)

- 1st digit: 2 choices (4, 6).
- 4th digit: 4 choices (1, 3, 5, 7 are all available).
- 2nd and 3rd digits: 5 and 4 choices left.
- Number of codes $= 2 \times 5 \times 4 \times 4 = 160$.

Case 2: First digit is odd (5 or 7)

- 1st digit: 2 choices (5, 7).
- 4th digit: 3 choices (one odd digit used at the front).
- 2nd and 3rd digits: 5 and 4 choices left.
- Number of codes $= 2 \times 5 \times 4 \times 3 = 120$.

Total $= 160 + 120 =$ **280**.

You Do 1

Available: $\{1, 2, 3, 4, 5\}$. 3 digits. > 300 means starts with 3, 4, 5. Even means ends in 2, 4. Overlap is 4.

Case 1: Starts with 3 or 5

- 1st digit: 2 choices (3, 5). Last digit: 2 choices (2, 4). Middle: 3 choices.
- Codes $= 2 \times 3 \times 2 = 12$.

Case 2: Starts with 4

- 1st digit: 1 choice (4). Last digit: 1 choice (2, since 4 is used). Middle: 3 choices.
- Codes $= 1 \times 3 \times 1 = 3$.

Total $= 12 + 3 =$ **15**.

You Do 2
Available: $\{2, 3, 4, 5, 6, 7, 8, 9\}$ (8 digits).
$< 5000 \implies$ starts with 2, 3, 4. Even $\implies$ ends with 2, 4, 6, 8. Overlap is 2, 4.
Case 1: Starts with odd (3)

- 1st digit: 1 choice (3). Last digit: 4 choices (2, 4, 6, 8). Middle two: 6×5.
- Codes $= 1 \times 6 \times 5 \times 4 = 120$.

Case 2: Starts with even (2 or 4)

- 1st digit: 2 choices. Last digit: 3 choices. Middle two: 6×5.
- Codes $= 2 \times 6 \times 5 \times 3 = 180$.

Total $= 120 + 180 =$ **300**.

You Do 3
Available: 10 digits. $> 70000 \implies$ starts with 7, 8, 9. Odd $\implies$ ends with 1, 3, 5, 7, 9. Overlap: 7, 9.
Case 1: Starts with even (8)

- 1st digit: 1 choice (8). Last digit: 5 choices. Middle three: $8 \times 7 \times 6$.
- Codes $= 1 \times 8 \times 7 \times 6 \times 5 = 1680$.

Case 2: Starts with odd (7 or 9)

- 1st digit: 2 choices. Last digit: 4 choices. Middle three: $8 \times 7 \times 6$.
- Codes $= 2 \times 8 \times 7 \times 6 \times 4 = 2688$.

Total $= 1680 + 2688 =$ **4368**.

Challenge 1
Available: $\{1, 2, 3, 4, 5, 6, 7\}$. Multiple of 5 means it MUST end in 5 (no 0 available).
So the last digit is fixed (1 choice: 5).
To be < 600, the first digit would normally be 1, 2, 3, 4, or 5.
However, no digit can be repeated, and 5 is already used for the last digit!
Therefore, the first digit can only be 1, 2, 3, or 4 (4 choices).
Middle digit: 5 choices remain from the 7 original digits.
Total $= 4 \times 5 \times 1 =$ **20**.

Challenge 2
Set: $\{1, 2, 3, 4, 5, 6\}$. 4 digits. $> 4000 \implies$ starts 4, 5, 6.
Multiple of 4 means the last two digits must be a multiple of 4. From the set without repetition, these pairs are: 12, 16, 24, 32, 36, 52, 56, 64.
We count based on each valid ending pair, remembering the 1st digit must be 4, 5, or 6 and cannot overlap with the ending pair.

- Ends in 12: 1st can be 4, 5, 6 (3 choices). Middle digit has 3 left. $3 \times 3 = 9$.
- Ends in 16: 1st can be 4, 5 (2 choices). Middle has 3 left. $2 \times 3 = 6$.
- Ends in 24: 1st can be 5, 6 (2 choices). Middle has 3 left. $2 \times 3 = 6$.
- Ends in 32: 1st can be 4, 5, 6 (3 choices). Middle has 3 left. $3 \times 3 = 9$.
- Ends in 36: 1st can be 4, 5 (2 choices). Middle has 3 left. $2 \times 3 = 6$.
- Ends in 52: 1st can be 4, 6 (2 choices). Middle has 3 left. $2 \times 3 = 6$.
- Ends in 56: 1st can be 4 (1 choice). Middle has 3 left. $1 \times 3 = 3$.
- Ends in 64: 1st can be 5 (1 choice). Middle has 3 left. $1 \times 3 = 3$.

Total $= 9 + 6 + 6 + 9 + 6 + 6 + 3 + 3 =$ **48**.

Extreme Challenge
Using all 6 digits $\{1, 2, 3, 4, 5, 6\}$. $> 300,000 \implies$ starts with 3, 4, 5, 6.
Odd $\implies$ ends with 1, 3, 5. Overlap is 3, 5.
Case 1: Starts with even (4 or 6)

- 1st digit: 2 choices. Last digit: 3 choices (1, 3, 5). Middle 4 digits: $4 \times 3 \times 2 \times 1 = 24$.
- Codes $= 2 \times 24 \times 3 = 144$.

Case 2: Starts with odd (3 or 5)

- 1st digit: 2 choices. Last digit: 2 choices left. Middle 4 digits: 24.
- Codes $= 2 \times 24 \times 2 = 96$.

Total $= 144 + 96 =$ **240**.