Quadratic Inequalities and Set Notation

Practice Worksheet (Edexcel 1MA1 Style)

I Do

Solve the inequality

$$3x^2 - 14x - 5 > 0$$

Write your answer using set notation.

. . . . . .

(Total for Question is 4 marks)

We Do

Solve the inequality

$$2x^2 + 7x - 15 \leq 0$$

Write your answer using set notation.

.....

(Total for Question is 4 marks)

You Do 1

Solve the inequality

$$x^2 - 2x - 24 > 0$$

Write your answer using set notation.

.....

(Total for Question is 3 marks)

You Do 2

Solve the inequality

$$4x^2 - 25 \leq 0$$

Write your answer using set notation.

.....

(Total for Question is 3 marks)

You Do 3

Solve the inequality

$$5x^2 - 13x - 6 < 0$$

Write your answer using set notation.

.....

(Total for Question is 4 marks)

Challenge 1

Find the set of values of x for which both of the following inequalities are true:

$$x^2 - 5x + 6 > 0$$

$$2x^2 - 7x - 15 \leq 0$$

Write your final answer using set notation.

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(Total for Question is 5 marks)

Challenge 2

A rectangle has a length of $(2x - 3)$ cm and a width of $(x + 4)$ cm.

The area of the rectangle must be at least $30\,\text{cm}^2$.

By forming and solving an inequality, find the set of possible values for x.
Write your answer using set notation.
Hint: Remember that the side lengths of a rectangle must be positive.

. . . . . .

(Total for Question is 5 marks)

Extreme Challenge

Solve the rational inequality

$$\frac{x^2 - 4x - 12}{x + 5} \geq 0$$

Write your answer using set notation.
Hint: You must consider when the numerator and denominator share the same sign, and be careful with the boundary condition at $x = -5$.

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(Total for Question is 5 marks)

Worked Answers

I Do

First, factorise the quadratic $3x^2 - 14x - 5$.
We need two numbers that multiply to $(3 \times -5) = -15$ and add to -14. These are -15 and 1.
Split the middle term: $3x^2 - 15x + x - 5 = 3x(x-5) + 1(x-5) = (3x+1)(x-5)$.
The critical values are where $(3x+1)(x-5) = 0$, so $x = -\frac{1}{3}$ and $x = 5$.
The parabola $y = 3x^2 - 14x - 5$ is U-shaped (positive x^2).
We want where the curve is > 0 (above the x-axis), which is the outside regions.
So, $x < -\frac{1}{3}$ or $x > 5$.
In set notation: $\{\mathbf{x} : \mathbf{x} < \mathbf{-\frac{1}{3}}\} \cup \{\mathbf{x} : \mathbf{x} > \mathbf{5}\}$.

We Do

Factorise $2x^2 + 7x - 15$.
Multiply to $(2 \times -15) = -30$, add to 7. The numbers are 10 and -3.
$2x^2 + 10x - 3x - 15 = 2x(x+5) - 3(x+5) = (2x-3)(x+5)$.
Critical values are $x = \frac{3}{2}$ and $x = -5$.
The parabola is U-shaped, and we want ≤ 0 (below or on the x-axis). This is the inner region.
$-5 \leq x \leq 1.5$.
In set notation: $\{\mathbf{x} : \mathbf{-5 \leq x \leq 1.5}\}$.

You Do 1

Factorise $x^2 - 2x - 24 > 0$.
$(x-6)(x+4) > 0$.
Critical values are $x = 6$ and $x = -4$.
U-shaped parabola, looking for > 0 means the outside regions.
$x < -4$ or $x > 6$.
In set notation: $\{\mathbf{x} : \mathbf{x} < \mathbf{-4}\} \cup \{\mathbf{x} : \mathbf{x} > \mathbf{6}\}$.

You Do 2

This is a difference of two squares: $4x^2 - 25 \leq 0 \implies (2x-5)(2x+5) \leq 0$.
Critical values are $x = 2.5$ and $x = -2.5$.
U-shaped parabola, looking for ≤ 0 means the inner region.
$-2.5 \leq x \leq 2.5$.
In set notation: $\{\mathbf{x} : \mathbf{-2.5 \leq x \leq 2.5}\}$.

You Do 3

Factorise $5x^2 - 13x - 6 < 0$.
Multiply to -30, add to -13. The numbers are -15 and 2.
$5x^2 - 15x + 2x - 6 = 5x(x-3) + 2(x-3) = (5x+2)(x-3) < 0$.
Critical values are $x = -0.4$ and $x = 3$.
Inner region required: $-0.4 < x < 3$.
In set notation: $\{\mathbf{x} : \mathbf{-0.4 < x < 3}\}$.

Challenge 1

Solve the first inequality:
$x^2 - 5x + 6 > 0 \implies (x-2)(x-3) > 0 \implies x < 2$ or $x > 3$.
Solve the second inequality:
$2x^2 - 7x - 15 \leq 0 \implies (2x+3)(x-5) \leq 0 \implies -1.5 \leq x \leq 5$.
We need the intersection where both conditions are met.
Plotting these on a number line, we want the overlapping regions of $[-1.5, 5]$ with $(-\infty, 2) \cup (3, \infty)$.
The overlap happens from -1.5 to 2 (not including 2) and from 3 (not including 3) up to 5.
$-1.5 \leq x < 2$ or $3 < x \leq 5$.
In set notation: $\{\mathbf{x} : \mathbf{-1.5 \leq x < 2}\} \cup \{\mathbf{x} : \mathbf{3 < x \leq 5}\}$.

Challenge 2

Area condition: $(2x-3)(x+4) \geq 30$.
Expand: $2x^2 + 8x - 3x - 12 \geq 30 \implies 2x^2 + 5x - 42 \geq 0$.
Factorise: $(2x-7)(x+6) \geq 0$.
Critical values are $x = 3.5$ and $x = -6$.
Regions: $x \leq -6$ or $x \geq 3.5$.
However, side lengths must be positive.
$2x - 3 > 0 \implies x > 1.5$.
$x + 4 > 0 \implies x > -4$.
Combining these practical constraints ($x > 1.5$) with our quadratic solution, the only valid region is $x \geq 3.5$.
In set notation: $\{\mathbf{x} : \mathbf{x \geq 3.5}\}$.

Extreme Challenge

Numerator: $x^2 - 4x - 12 = (x-6)(x+2)$. The roots are $x = 6$ and $x = -2$.
Denominator: $x + 5$. The root is $x = -5$. (Note: x cannot exactly equal -5 as we cannot divide by zero).
The critical values that partition the number line are $-5, -2$, and 6.
Check intervals:
$x < -5$ (e.g. $x = -6$): $\frac{(-)(-)}{-} =$ negative
$-5 < x < -2$ (e.g. $x = -3$): $\frac{(-)(-)}{+} =$ positive
$-2 < x < 6$ (e.g. $x = 0$): $\frac{(-)(+)}{+} =$ negative
$x > 6$ (e.g. $x = 7$): $\frac{(+)(+)}{+} =$ positive
We want ≥ 0 (positive or zero). The boundary values $x = -2$ and $x = 6$ make the expression 0 (which is valid), but $x = -5$ must be strictly excluded.
Valid regions: $-5 < x \leq -2$ or $x \geq 6$.
In set notation: $\{\mathbf{x} : \mathbf{-5 < x \leq -2}\} \cup \{\mathbf{x} : \mathbf{x \geq 6}\}$.