Histograms: Frequency Density and Median

Practice Worksheet (Edexcel 1MA1 Style)

I Do

The incomplete table gives information about the weights, in kilograms, of 160 luggage bags at an airport check-in desk.

Weight (w kg)	**Frequency**
$0 < w \leq 10$	30
$10 < w \leq 15$	a
$15 < w \leq 25$	60
$25 < w \leq 40$	b

On a histogram drawn to represent this data:

- the bar representing the class $0 < w \leq 10$ has a height of 6 cm on the vertical (frequency density) scale.
- the height of the bar for $10 < w \leq 15$ is twice the height of the bar for $25 < w \leq 40$.

(a) Work out the value of a and the value of b.

$a =$ ....., $b =$ ..... **(3)**

(b) Write the class interval which contains the median.

..... **(1)**

(Total for Question is 4 marks)

We Do

The incomplete table gives information about the weights, in grams, of 120 parcels.

Weight (w g)	**Frequency**
$0 < w \leq 10$	20
$10 < w \leq 20$	a
$20 < w \leq 30$	40
$30 < w \leq 50$	b

On a histogram drawn to represent this data:

- the bar representing the class $0 < w \leq 10$ has a height of 4 cm on the vertical scale.
- the height of the bar for $10 < w \leq 20$ is exactly 3 times the height of the bar for $30 < w \leq 50$.

(a) Work out the value of a and the value of b.

$a =$ ....., $b =$ ..... **(3)**

(b) Write the class interval which contains the median.

..... **(1)**

(Total for Question is 4 marks)

You Do 1

The incomplete table gives information about the times, in minutes, taken by 200 athletes to complete a race.

Time (t mins)	**Frequency**
$0 < t \leq 10$	40
$10 < t \leq 25$	x
$25 < t \leq 40$	75
$40 < t \leq 60$	y

On a histogram drawn to represent this data:

- the bar representing the class $0 < t \leq 10$ has a height of 8 cm on the vertical scale.
- the height of the bar for $10 < t \leq 25$ is twice the height of the bar for $40 < t \leq 60$.

(a) Work out the value of x and the value of y.

$x =$ ....., $y =$ ..... **(3)**

(b) Write the class interval which contains the median.

..... **(1)**

(Total for Question is 4 marks)

You Do 2

The incomplete table gives information about the masses, in kg, of 150 dogs.

Mass (m kg)	**Frequency**
$0 < m \leq 15$	p
$15 < m \leq 20$	q
$20 < m \leq 30$	40
$30 < m \leq 50$	30

On a histogram drawn to represent this data:

- the bar representing the class $20 < m \leq 30$ has a height of 12 cm on the vertical scale.
- the height of the bar for $0 < m \leq 15$ is exactly the same as the height of the bar for $15 < m \leq 20$.

(a) Work out the value of p and the value of q.

$p =$ ....., $q =$ ..... **(3)**

(b) Write the class interval which contains the median.

..... **(1)**

(Total for Question is 4 marks)

You Do 3

The incomplete table gives information about the lengths, in cm, of 100 fish in a pond.

Length (L cm)	**Frequency**
$0 < L \leq 10$	15
$10 < L \leq 15$	c
$15 < L \leq 20$	d
$20 < L \leq 40$	10

On a histogram drawn to represent this data:

- the bar representing the class $0 < L \leq 10$ has a height of 1.5 cm on the vertical scale.
- the height of the bar for $10 < L \leq 15$ is twice the height of the bar for $15 < L \leq 20$.

(a) Work out the value of c and the value of d.

$c =$ ....., $d =$ ..... **(3)**

(b) Write the class interval which contains the median.

..... **(1)**

(Total for Question is 4 marks)

Challenge 1

The incomplete table gives information about the scores of 230 students on a test.

Score (s)	**Frequency**
$0 < s \leq 10$	50
$10 < s \leq 20$	a
$20 < s \leq 30$	b
$30 < s \leq 50$	c

On a histogram drawn to represent this data:

- the heights of the bars for the classes $10 < s \leq 20$, $20 < s \leq 30$, and $30 < s \leq 50$ are in the ratio $4 : 3 : 1$ respectively.

(a) Find the values of a, b, and c.

$a =$ ....., $b =$ ....., $c =$ ..... **(4)**

(b) Write the class interval which contains the median.

..... **(1)**

(Total for Question is 5 marks)

Challenge 2

The incomplete table gives information about the daily wages, in £, of some workers.

Wage (£w)	**Frequency**
$0 < w \leq 10$	20
$10 < w \leq 20$	$3x$
$20 < w \leq 40$	$x + 10$
$40 < w \leq 70$	15

On a histogram drawn to represent this data, the height of the bar for the class $10 < w \leq 20$ is exactly 4 times the height of the bar for the class $20 < w \leq 40$.

(a) Find the exact total number of workers.
You must show all your algebraic working.

. . . . . **(4)**

(b) Write the class interval which contains the median.

£. . . . . **(1)**

(Total for Question is 5 marks)

Extreme Challenge

A histogram is drawn to represent the ages, in years, of some people at a concert.
The data is grouped into the following classes:

Age (A years)	**Frequency**
$0 < A \leq 10$	10
$10 < A \leq 20$	30
$20 < A \leq 30$	x
$30 < A \leq 50$	y

You are given the following two pieces of information:

- The median age of the people at the concert is exactly 25 years.
- On the histogram, the height of the bar for $20 < A \leq 30$ is equal to the height of the bar for $30 < A \leq 50$.

Find the total number of people at the concert.
You must show all your working clearly.

.....

(Total for Question is 5 marks)

Worked Answers

I Do

(a) First, find the scale factor for the histogram height.
For $0 < w \leq 10$: Class Width (CW) $= 10$. Frequency $= 30$.
Frequency Density (FD) $= 30 \div 10 = 3$.
The height is 6 cm, so Height $= 2 \times$ FD.
Now use this relationship for the other two classes:
For $10 < w \leq 15$: CW $= 5$, FD $= \frac{a}{5}$. Height $= 2 \times \frac{a}{5} = \frac{2a}{5}$.
For $25 < w \leq 40$: CW $= 15$, FD $= \frac{b}{15}$. Height $= 2 \times \frac{b}{15} = \frac{2b}{15}$.
We are told the height for $10 - 15$ is twice the height for $25 - 40$:
$\frac{2a}{5} = 2 \times \left(\frac{2b}{15}\right) \implies \frac{2a}{5} = \frac{4b}{15} \implies 6a = 4b \implies 3a = 2b \implies b = 1.5a$.
We know the total frequency is 160:
$30 + a + 60 + b = 160 \implies a + b = 70$.
Substitute $b = 1.5a$:
$a + 1.5a = 70 \implies 2.5a = 70 \implies \mathbf{a = 28}$.
$b = 1.5 \times 28 = \mathbf{42}$.

(b) The median position is $160 \div 2 = 80$.
Cumulative frequencies: 30, 58, 118, 160.
The class interval which contains the median is $\mathbf{15 < w \leq 25}$.

We Do

(a) Total $= 120$. So $20 + a + 40 + b = 120 \implies a + b = 60$.
For $0 < w \leq 10$: CW $= 10$, Freq $= 20$, FD $= 20 \div 10 = 2$.
Height is 4 cm, so Height $= 2 \times$ FD.
For $10 < w \leq 20$: CW $= 10$, Height $= 2 \times \frac{a}{10} = \frac{a}{5}$.
For $30 < w \leq 50$: CW $= 20$, Height $= 2 \times \frac{b}{20} = \frac{b}{10}$.
Height ratio: $\frac{a}{5} = 3 \times \left(\frac{b}{10}\right) \implies 2a = 3b \implies a = 1.5b$.
Substitute into $a + b = 60$:
$1.5b + b = 60 \implies 2.5b = 60 \implies \mathbf{b = 24}$.
$a = 1.5 \times 24 = \mathbf{36}$.

(b) The median position is $120 \div 2 = 60$.
Cumulative frequencies: 20, 56, 96, 120.
The class interval which contains the median is $\mathbf{20 < w \leq 30}$.

Median class intervals: B1 (1 mark) for each correct interval.

You Do 1

(a) Total = 200. $40 + x + 75 + y = 200 \implies x + y = 85$.
$0 < t \leq 10$: CW = 10, FD = 4. Height = 8 cm, so Height = 2 × FD.
$10 < t \leq 25$: CW = 15, Height = $2 \times \frac{x}{15} = \frac{2x}{15}$.
$40 < t \leq 60$: CW = 20, Height = $2 \times \frac{y}{20} = \frac{y}{10}$.
Height ratio: $\frac{2x}{15} = 2 \times \left(\frac{y}{10}\right) \implies \frac{2x}{15} = \frac{y}{5} \implies 2x = 3y \implies x = 1.5y$.
$1.5y + y = 85 \implies 2.5y = 85 \implies \mathbf{y = 34}$.
$x = 1.5 \times 34 = \mathbf{51}$.

(b) The median position is 200 ÷ 2 = 100.
Cumulative frequencies: 40, 91, 166, 200.
The class interval which contains the median is **25 < t ≤ 40**.

You Do 2

(a) Total = 150. $p + q + 40 + 30 = 150 \implies p + q = 80$.
$20 < m \leq 30$: CW = 10, Freq = 40, FD = 4. Height = 12 cm, so Height = 3 × FD.
$0 < m \leq 15$: CW = 15, Height = $3 \times \frac{p}{15} = \frac{p}{5}$.
$15 < m \leq 20$: CW = 5, Height = $3 \times \frac{q}{5} = \frac{3q}{5}$.
Height ratio: $\frac{p}{5} = \frac{3q}{5} \implies p = 3q$.
$3q + q = 80 \implies 4q = 80 \implies \mathbf{q = 20}$.
$p = 3 \times 20 = \mathbf{60}$.

(b) The median position is 150 ÷ 2 = 75.
Cumulative frequencies: 60, 80, 120, 150.
The class interval which contains the median is **15 < m ≤ 20**.

You Do 3

(a) Total = 100. $15 + c + d + 10 = 100 \implies c + d = 75$.
$0 < L \leq 10$: CW = 10, Freq = 15, FD = 1.5. Height = 1.5 cm, so Height = 1 × FD.
$10 < L \leq 15$: CW = 5, Height = $\frac{c}{5}$.
$15 < L \leq 20$: CW = 5, Height = $\frac{d}{5}$.
Height ratio: $\frac{c}{5} = 2 \times \frac{d}{5} \implies c = 2d$.
$2d + d = 75 \implies 3d = 75 \implies \mathbf{d = 25}$.
$c = 2 \times 25 = \mathbf{50}$.

(b) The median position is 100 ÷ 2 = 50.
Cumulative frequencies: 15, 65, 90, 100.
The class interval which contains the median is **10 < L ≤ 15**.

Challenge 1

(a) Let the vertical scale factor be k so Height $= k \times$ FD.
Since height ratios are $4:3:1$, the FD ratios are also $4:3:1$.
$FD_a = \frac{a}{10}$, $FD_b = \frac{b}{10}$, $FD_c = \frac{c}{20}$.
$\frac{a}{10} : \frac{b}{10} : \frac{c}{20} = 4:3:1$.
Let $\frac{c}{20} = m \implies c = 20m$.
Then $\frac{b}{10} = 3m \implies b = 30m$.
And $\frac{a}{10} = 4m \implies a = 40m$.
Total frequency $= 50 + 40m + 30m + 20m = 230 \implies 90m = 180 \implies m = 2$.
$a = 40(2) = \mathbf{80}$, $b = 30(2) = \mathbf{60}$, $c = 20(2) = \mathbf{40}$.

(b) The median position is $230 \div 2 = 115$.
Cumulative frequencies: 50, 130, 190, 230.
The class interval which contains the median is $\mathbf{10 < s \leq 20}$.

Challenge 2

(a) Height ratio between $10 < w \leq 20$ and $20 < w \leq 40$ is 4:1.
$FD_1 = \frac{3x}{10}$. $FD_2 = \frac{x+10}{20}$.
$\frac{3x}{10} = 4 \times \left(\frac{x+10}{20}\right) \implies \frac{3x}{10} = \frac{2(x+10)}{10}$.
Multiply by 10: $3x = 2x + 20 \implies x = 20$.
The frequencies are: $20, 3(20) = 60, 20 + 10 = 30, 15$.
Total workers $= 20 + 60 + 30 + 15 = \mathbf{125}$.

(b) The median position is $125 \div 2 = 62.5$.
Cumulative frequencies: 20, 80, 110, 125.
The class interval which contains the median is $\mathbf{10 < w \leq 20}$.

Extreme Challenge

The median age is exactly 25. This is exactly in the middle of the class $20 < A \leq 30$.
Because the median falls exactly halfway through the class interval, exactly half of the frequency x must lie below the median, and half above. This implies that the total number of people younger than 20 plus half of x equals the total number of people older than 30 plus half of x.
Number below $20 = 10 + 30 = 40$. Number above $30 = y$.
Therefore, $40 + 0.5x = y + 0.5x \implies y = 40$.
We are also told the height of the bar for $20 - 30$ equals the height of the bar for $30 - 50$.
$FD_{20-30} = \frac{x}{10}$. $FD_{30-50} = \frac{y}{20} = \frac{40}{20} = 2$.
$\frac{x}{10} = 2 \implies x = 20$.
Total number of people $= 10 + 30 + x + y = 10 + 30 + 20 + 40 = \mathbf{100}$.