Venn Diagrams and Algebraic Probabilities

Practice Worksheet (Edexcel 1MA1 Style)

I Do

ξ is the universal set of 100 students in a year group.

A and B are two clubs that students can join.

- x^2 students belong to club A only.
- $5x$ students belong to both club A and club B.
- $2x + 10$ students belong to club B only.
- 12 students do not belong to either club.

A student is chosen at random from the year group.
Work out the probability that this student belongs to club A.

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(Total for Question is 4 marks)

We Do

Catherine surveys 80 students in her year group about two clubs, P and Q. ξ is the universal set of these 80 students.

- x^2 students belong to club P only.
- $3x$ students belong to both club P and club Q.
- $3x + 8$ students belong to club Q only.
- 17 students do not belong to either club.

A student is chosen at random from the 80 students.
Work out the probability that this student belongs to club Q.

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(Total for Question is 4 marks)

You Do 1

John gathers data on 138 students regarding two sports teams, X and Y. ξ is the universal set of these 138 students.

- x^2 students belong to team X only.
- $4x$ students belong to both team X and team Y.
- $5x - 2$ students belong to team Y only.
- 28 students do not belong to either team.

A student is chosen at random.
Work out the probability that this student **does not** belong to team X.

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(Total for Question is 4 marks)

You Do 2

Ryan is organising a trip for 105 students. They can choose to do Archery (A) or Bouldering (B).

- $2x^2$ students choose Archery only.
- x students choose both Archery and Bouldering.
- $4x + 10$ students choose Bouldering only.
- 20 students choose neither activity.

A student is chosen at random.
Work out the probability that this student chose at least one of the activities.

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(Total for Question is 4 marks)

You Do 3

Paul collects information from 52 adults about whether they own a Cat (C) or a Dog (D).

- $x^2 - 1$ adults own a Cat only.
- $2x + 3$ adults own both a Cat and a Dog.
- x^2 adults own a Dog only.
- 10 adults own neither.

An adult is chosen at random.
Work out the probability that this adult owns exactly one of these pets.

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(Total for Question is 4 marks)

Challenge 1

Sophie surveys 150 students about two music genres they listen to: Jazz (J) and Classical (C).

- x^2 students listen to Jazz only.
- $3x$ students listen to both Jazz and Classical.
- y students listen to Classical only.
- $2y$ students do not listen to either.

The ratio of the number of students who listen to Jazz only to the number of students who listen to both is $2 : 1$.

A student is chosen at random.
Work out the probability that this student listens to Classical music.
Give your answer as a fraction in its simplest form.

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(Total for Question is 5 marks)

Challenge 2

Paige looks at data for 75 members of a gym regarding swimming (S) and weightlifting (W).

- x^2 members do swimming only.
- xy members do both swimming and weightlifting.
- y^2 members do weightlifting only.
- 14 members do neither.

The total number of members who do swimming is 36.
Both x and y are positive integers.

A gym member is chosen at random.
Work out the probability that this member does exactly one of the two activities.

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(Total for Question is 6 marks)

Extreme Challenge

Katie analyses the language subjects taken by 168 students. The subjects are French (F), Spanish (S), and German (G).

- $2x^2$ students study French only.
- $3x^2$ students study Spanish only.
- x^2 students study German only.
- $4x$ students study exactly French and Spanish.
- $4x$ students study exactly French and German.
- $4x$ students study exactly Spanish and German.
- x students study all three languages.
- $5x$ students study none of these languages.

A student is chosen at random from the 168 students.
Find the probability that this student studies at least two languages.

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(Total for Question is 5 marks)

Worked Answers

I Do

The sum of all the regions in the Venn diagram must equal the total number of students, 100.
$x^2 + 5x + (2x + 10) + 12 = 100$
$x^2 + 7x + 22 = 100$
Subtract 100 from both sides:
$x^2 + 7x - 78 = 0$
Factorise the quadratic. We need two numbers that multiply to -78 and add to 7. These are 13 and -6.
$(x + 13)(x - 6) = 0$
So $x = -13$ or $x = 6$. Since x represents a number of students, x must be a positive number.
So $x = 6$.
The number of students in club A is the sum of "A only" and "both":
Club $A = x^2 + 5x = 6^2 + 5(6) = 36 + 30 = 66$.
Probability $= \frac{66}{100} = \mathbf{\frac{33}{50}}$.

We Do

Set up the equation for the total number of students:
$x^2 + 3x + (3x + 8) + 17 = 80$
$x^2 + 6x + 25 = 80$
$x^2 + 6x - 55 = 0$
Factorise: $(x + 11)(x - 5) = 0$. Because a frequency must be positive, $x = 5$.
The number of students in club Q is "Q only" plus "both":
Club $Q = (3x + 8) + 3x = 6x + 8$.
Using $x = 5$: Club $Q = 6(5) + 8 = 30 + 8 = 38$.
Probability $= \frac{38}{80} = \mathbf{\frac{19}{40}}$.

You Do 1

Total students = 138.
$x^2 + 4x + (5x - 2) + 28 = 138$
$x^2 + 9x + 26 = 138 \implies x^2 + 9x - 112 = 0$
Factorise: $(x + 16)(x - 7) = 0 \implies x = 7$ (since $x > 0$).
Students in team $X = x^2 + 4x = 7^2 + 4(7) = 49 + 28 = 77$.
Students **not** in team $X = 138 - 77 = 61$.
Probability $= \mathbf{\frac{61}{138}}$.

You Do 2

Total students = 105.
$2x^2 + x + (4x + 10) + 20 = 105$
$2x^2 + 5x + 30 = 105 \implies 2x^2 + 5x - 75 = 0$
Factorise: $(2x + 15)(x - 5) = 0 \implies x = 5$.
"At least one" is the total inside the circles, which is $105 - 20 = 85$.
(Alternatively, $2(5^2) + 5 + 4(5) + 10 = 50 + 5 + 30 = 85$).
Probability $= \frac{85}{105} = \mathbf{\frac{17}{21}}$.

You Do 3

Total adults = 52.
$(x^2 - 1) + (2x + 3) + x^2 + 10 = 52$
$2x^2 + 2x + 12 = 52 \implies 2x^2 + 2x - 40 = 0$
Divide by 2: $x^2 + x - 20 = 0 \implies (x + 5)(x - 4) = 0 \implies x = 4$.
"Exactly one" means C only + D only:
C only $= 4^2 - 1 = 15$. D only $= 4^2 = 16$. Total $= 15 + 16 = 31$.
Probability $= \mathbf{\frac{31}{52}}$.

Challenge 1

Ratio of Jazz only to both is 2 : 1.
$\frac{x^2}{3x} = 2 \implies x^2 = 6x \implies x(x - 6) = 0$. Since $x > 0$, $x = 6$.
Number of students doing Jazz only $= 6^2 = 36$. Both $= 3(6) = 18$.
Total students = 150.
$36 + 18 + y + 2y = 150 \implies 54 + 3y = 150 \implies 3y = 96 \implies y = 32$.
Number in Classical $= y$ (only) + Both $= 32 + 18 = 50$.
Probability $= \frac{50}{150} = \mathbf{\frac{1}{3}}$.

Challenge 2

Total members = 75. Subtract the 14 who do neither:
$x^2 + xy + y^2 = 75 - 14 = 61$.
Total in swimming = 36:
$x^2 + xy = 36$.
Substitute $x^2 + xy = 36$ into the first equation:
$36 + y^2 = 61 \implies y^2 = 25 \implies y = 5$ (since y is a positive integer).
Now solve for x:
$x^2 + 5x = 36 \implies x^2 + 5x - 36 = 0 \implies (x + 9)(x - 4) = 0 \implies x = 4$.
"Exactly one" means swimming only (x^2) + weightlifting only (y^2).
$4^2 + 5^2 = 16 + 25 = 41$.
Probability $= \mathbf{\frac{41}{75}}$.

Extreme Challenge

Form an equation for the total number of students:
$2x^2 + 3x^2 + x^2 + 4x + 4x + 4x + x + 5x = 168$
$6x^2 + 18x = 168$
Divide by 6:
$x^2 + 3x = 28 \implies x^2 + 3x - 28 = 0$
$(x + 7)(x - 4) = 0 \implies x = 4$.
We want the probability of "at least two languages". This is the sum of the intersection regions:
$(F \cap S$ only$) + (F \cap G$ only$) + (S \cap G$ only$) + (F \cap S \cap G)$
$= 4x + 4x + 4x + x = 13x$.
Substitute $x = 4$: $13 \times 4 = 52$ students.
Probability $= \frac{52}{168} = \mathbf{\frac{13}{42}}$.