Advanced Algebraic Fractions

Practice Worksheet (Problem Solving)

I Do

Express $\left(\frac{4-x}{3x+2} \div \frac{3x^2+10x-8}{18x^2-8}\right) - (2-x)$ as a single fraction in its simplest form.

Show clear algebraic working.

.....

(Total for Question is 5 marks)

We Do

Express $\left(\frac{5-x}{2x+3} \div \frac{2x^2+11x-21}{8x^2-18}\right) - (3-x)$ as a single fraction in its simplest form.

Show clear algebraic working.

.....

(Total for Question is 5 marks)

You Do 1

Express $\left(\frac{x+1}{4x-1} \div \frac{x^2-3x-4}{16x^2-1}\right) + 2$ as a single fraction in its simplest form.

Show clear algebraic working.

.....

(Total for Question is 4 marks)

You Do 2

Express $\left(\frac{x-2}{3x+1} \div \frac{x^2-4}{9x^2-1}\right) - \frac{2}{x+2}$ as a single fraction in its simplest form.

Show clear algebraic working.

.....

(Total for Question is 4 marks)

You Do 3

Express $\left(\frac{2x+3}{x-1} \div \frac{2x^2+x-3}{x^2-2x+1}\right) - \frac{x}{x+1}$ as a single fraction in its simplest form.

Show clear algebraic working.

.....

(Total for Question is 4 marks)

Challenge 1

Use algebra to show that

$$\frac{2x^2+5x-12}{4x^2-9} \div \frac{x^2+3x-4}{2x^2+5x+3} = \frac{x+1}{x-1}$$

.....

(Total for Question is 4 marks)

Challenge 2

Express $\frac{1}{x+2} - \frac{x-2}{2x^2+3x-2} \div \frac{x^2-4}{2x^2-x}$ as a single fraction in its simplest form.

Show clear algebraic working.

.....

(Total for Question is 5 marks)

Extreme Challenge

Solve the equation:

$$\frac{x^2 - 9}{2x^2 + 5x - 3} \div \frac{x^2 - 3x}{2x^2 - 7x + 3} = \frac{1}{2}$$

You must show all your algebraic working clearly.

$x =$ .....

(Total for Question is 5 marks)

Worked Answers

I Do

First, factorise all the quadratic expressions:
$3x^2 + 10x - 8 = (3x - 2)(x + 4)$
$18x^2 - 8 = 2(9x^2 - 4) = 2(3x - 2)(3x + 2)$
Rewrite the division as multiplication by the reciprocal:
$\frac{4-x}{3x+2} \times \frac{2(3x-2)(3x+2)}{(3x-2)(x+4)}$
Cancel the common factors $(3x + 2)$ and $(3x - 2)$:
$= \frac{2(4-x)}{x+4} = \frac{8-2x}{x+4}$
Now subtract $(2 - x)$:
$\frac{8-2x}{x+4} - \frac{(2-x)(x+4)}{x+4}$
Expand the numerator of the second fraction:
$(2 - x)(x + 4) = 2x + 8 - x^2 - 4x = -x^2 - 2x + 8$
Subtract this entire expression:
$\frac{8-2x-(-x^2-2x+8)}{x+4} = \frac{8-2x+x^2+2x-8}{x+4}$
Simplify the numerator to get the final answer:
$\mathbf{\frac{x^2}{x+4}}$

We Do

Factorise the quadratics:
$2x^2 + 11x - 21 = (2x - 3)(x + 7)$
$8x^2 - 18 = 2(4x^2 - 9) = 2(2x - 3)(2x + 3)$
Rewrite division as multiplication:
$\frac{5-x}{2x+3} \times \frac{2(2x-3)(2x+3)}{(2x-3)(x+7)}$
Cancel the common factors $(2x + 3)$ and $(2x - 3)$:
$= \frac{2(5-x)}{x+7} = \frac{10-2x}{x+7}$
Now subtract $(3 - x)$:
$\frac{10-2x}{x+7} - \frac{(3-x)(x+7)}{x+7}$
Expand $(3 - x)(x + 7) = 3x + 21 - x^2 - 7x = -x^2 - 4x + 21$
Subtract:
$\frac{10-2x-(-x^2-4x+21)}{x+7} = \frac{10-2x+x^2+4x-21}{x+7}$
$\mathbf{\frac{x^2+2x-11}{x+7}}$

You Do 1

Factorise:
$x^2 - 3x - 4 = (x - 4)(x + 1)$
$16x^2 - 1 = (4x - 1)(4x + 1)$
Rewrite and multiply:
$\frac{x+1}{4x-1} \times \frac{(4x-1)(4x+1)}{(x-4)(x+1)}$
Cancel $(4x - 1)$ and $(x + 1)$:
$= \frac{4x+1}{x-4}$
Now add 2:
$\frac{4x+1}{x-4} + \frac{2(x-4)}{x-4} = \frac{4x+1+2x-8}{x-4}$
$\mathbf{\frac{6x-7}{x-4}}$

You Do 2

Factorise:

$x^2 - 4 = (x - 2)(x + 2)$

$9x^2 - 1 = (3x - 1)(3x + 1)$

Rewrite and multiply:

$\frac{x-2}{3x+1} \times \frac{(3x-1)(3x+1)}{(x-2)(x+2)}$

Cancel $(x - 2)$ and $(3x + 1)$:

$= \frac{3x-1}{x+2}$

Now subtract $\frac{2}{x+2}$:

$\frac{3x-1}{x+2} - \frac{2}{x+2} = \frac{3x-1-2}{x+2}$

$\mathbf{\frac{3x-3}{x+2}}$ (or $\mathbf{\frac{3(x-1)}{x+2}}$)

You Do 3

Factorise:

$2x^2 + x - 3 = (2x + 3)(x - 1)$

$x^2 - 2x + 1 = (x - 1)^2$

Rewrite and multiply:

$\frac{2x+3}{x-1} \times \frac{(x-1)^2}{(2x+3)(x-1)}$

Cancel $(2x + 3)$ and $(x - 1)^2$:

$= 1$

Now subtract $\frac{x}{x+1}$:

$1 - \frac{x}{x+1} = \frac{x+1}{x+1} - \frac{x}{x+1}$

$\mathbf{\frac{1}{x+1}}$

Challenge 1

Factorise all four expressions:

$2x^2 + 5x - 12 = (2x - 3)(x + 4)$

$4x^2 - 9 = (2x - 3)(2x + 3)$

$x^2 + 3x - 4 = (x + 4)(x - 1)$

$2x^2 + 5x + 3 = (2x + 3)(x + 1)$

Rewrite the division as multiplication:

$\frac{(2x-3)(x+4)}{(2x-3)(2x+3)} \times \frac{(2x+3)(x+1)}{(x+4)(x-1)}$

Cancel common factors:

$(2x - 3)$ cancels top and bottom on the first fraction.

$(x + 4)$ cancels across the fractions.

$(2x + 3)$ cancels across the fractions.

The only terms remaining are $(x + 1)$ on the top and $(x - 1)$ on the bottom.

$= \mathbf{\frac{x+1}{x-1}}$ (as required)

Challenge 2

According to BIDMAS, do the division first.

Factorise the expressions for the division part:

$2x^2 + 3x - 2 = (2x - 1)(x + 2)$

$x^2 - 4 = (x - 2)(x + 2)$

$2x^2 - x = x(2x - 1)$

Rewrite the division as multiplication:

$\frac{x-2}{(2x-1)(x+2)} \times \frac{x(2x-1)}{(x-2)(x+2)}$

Cancel $(x - 2)$ and $(2x - 1)$:

$= \frac{x}{(x+2)^2}$

Now substitute this back into the original subtraction:

$\frac{1}{x+2} - \frac{x}{(x+2)^2}$

Find a common denominator of $(x + 2)^2$:

$\frac{x+2}{(x+2)^2} - \frac{x}{(x+2)^2} = \frac{x+2-x}{(x+2)^2}$

$\mathbf{\frac{2}{(x+2)^2}}$

Extreme Challenge

Simplify the algebraic fraction first.

Factorise all expressions:

$x^2 - 9 = (x - 3)(x + 3)$

$2x^2 + 5x - 3 = (2x - 1)(x + 3)$

$x^2 - 3x = x(x - 3)$

$2x^2 - 7x + 3 = (2x - 1)(x - 3)$

Rewrite as multiplication:

$\frac{(x-3)(x+3)}{(2x-1)(x+3)} \times \frac{(2x-1)(x-3)}{x(x-3)}$

Cancel $(x + 3)$ in the first fraction.

Cancel $(x - 3)$ in the second fraction.

Cancel $(2x - 1)$ across the fractions.

The entire left-hand side simplifies to:

$\frac{x-3}{x}$

Set this equal to $\frac{1}{2}$:

$\frac{x-3}{x} = \frac{1}{2}$

Cross-multiply to solve:

$2(x - 3) = 1(x)$

$2x - 6 = x$

Subtract x and add 6:

$\mathbf{x = 6}$