Advanced Indices and Prime Bases

Practice Worksheet (Grade 9 Mastery)

I Do

Find the values of n such that

$$\left(\frac{10^{2n} \times 2^{2(n^2-4n)} \times 5^{2(2-n)}}{50^2}\right)^{\frac{2}{3}} = 16$$

Show clear algebraic working.

. . . . . .

(Total for Question is 5 marks)

Video walkthrough

We Do

Find the values of n such that

$$\left(\frac{6^{2n} \times 2^{n^2-4n} \times 3^{4-2n}}{81}\right)^{\frac{1}{3}} = 2$$

Show clear algebraic working.

. . . . . .

(Total for Question is 5 marks)

You Do 1

Find the values of n such that

$$\frac{14^n \times 2^{n^2-2n} \times 7^{2-n}}{49} = 64$$

Show clear algebraic working.

.....

(Total for Question is 4 marks)

You Do 2

Find the values of n such that

$$\frac{15^{3n} \times 5^{2n^2+2n} \times 3^{2-3n}}{9} = 125$$

Show clear algebraic working.

.....

(Total for Question is 5 marks)

You Do 3

Find the values of n such that

$$\left(\frac{21^n \times 7^{n^2-3n} \times 3^{1-n}}{3}\right)^{-1} = \frac{1}{343}$$

Show clear algebraic working.

.....

(Total for Question is 4 marks)

Challenge 1

Find the values of x such that

$$\frac{27^x \times 3^{x^2}}{9^{2x}} = 729$$

Show clear algebraic working.

.....

(Total for Question is 4 marks)

Challenge 2

Find the exact values of x and y that satisfy both of the following equations:

$$2^x \times 4^y = 512$$

$$\frac{3^{x^2}}{9^y} = 27$$

You are given that $x > 0$.
Show clear algebraic working.

$x =$ ....., $y =$ .....

(Total for Question is 5 marks)

Extreme Challenge

Find the exact values of x such that

$$\frac{12^x \times 18^{x^2} \times 2^{4-2x}}{3^{2x^2+x} \times 4^{x^2}} = 0.125$$

You must show all your algebraic working clearly.

.....

(Total for Question is 5 marks)

Worked Answers

I Do

First, express all bases as products of their prime factors (2 and 5):
$10^{2n} = (2 \times 5)^{2n} = 2^{2n} \times 5^{2n}$
$50^2 = (2 \times 25)^2 = (2 \times 5^2)^2 = 2^2 \times 5^4$
Now substitute these into the numerator and denominator inside the bracket:
Numerator $= (2^{2n} \times 5^{2n}) \times 2^{2n^2-8n} \times 5^{4-2n}$
Combine the powers of 2 and 5 in the numerator:
Base 2: $2^{2n} \times 2^{2n^2-8n} = 2^{2n^2-6n}$
Base 5: $5^{2n} \times 5^{4-2n} = 5^4$
The expression inside the bracket is now:
$\frac{2^{2n^2-6n} \times 5^4}{2^2 \times 5^4}$
The 5^4 terms cancel out. Subtract the power of the denominator for base 2:
2^{2n^2-6n-2}
Apply the outer exponent:
$\left(2^{2n^2-6n-2}\right)^{2/3} = 2^{\frac{2}{3}(2n^2-6n-2)}$
We are given that this equals 16, which is 2^4.
Equating the exponents:
$\frac{2}{3}(2n^2 - 6n - 2) = 4$
Multiply by 3 and divide by 2:
$2n^2 - 6n - 2 = 6 \implies 2n^2 - 6n - 8 = 0$
Divide by 2: $n^2 - 3n - 4 = 0$
Factorise: $(n - 4)(n + 1) = 0$
$\mathbf{n = 4}$ or $\mathbf{n = -1}$.

We Do

Prime factorise the bases:
$6^{2n} = (2 \times 3)^{2n} = 2^{2n} \times 3^{2n}$
$81 = 3^4$
Substitute into the numerator:
Base 2: $2^{2n} \times 2^{n^2-4n} = 2^{n^2-2n}$
Base 3: $3^{2n} \times 3^{4-2n} = 3^4$
Expression inside bracket: $\frac{2^{n^2-2n} \times 3^4}{3^4} = 2^{n^2-2n}$
Apply the outer exponent:
$\left(2^{n^2-2n}\right)^{1/3} = 2^{\frac{1}{3}(n^2-2n)}$
Equate to 2 (which is 2^1):
$\frac{1}{3}(n^2 - 2n) = 1 \implies n^2 - 2n = 3 \implies n^2 - 2n - 3 = 0$
$(n - 3)(n + 1) = 0$
$\mathbf{n = 3}$ or $\mathbf{n = -1}$.

You Do 1

Prime bases: $14^n = 2^n \times 7^n$, and $49 = 7^2$.
Numerator:
Base 2: $2^n \times 2^{n^2-2n} = 2^{n^2-n}$
Base 7: $7^n \times 7^{2-n} = 7^2$
Expression: $\frac{2^{n^2-n} \times 7^2}{7^2} = 2^{n^2-n}$
Set equal to 64 (which is 2^6):
$2^{n^2-n} = 2^6 \implies n^2 - n = 6 \implies n^2 - n - 6 = 0$
$(n-3)(n+2) = 0 \implies \mathbf{n = 3}$ or $\mathbf{n = -2}$.

You Do 2

Prime bases: $15^{3n} = 3^{3n} \times 5^{3n}$, and $9 = 3^2$.
Numerator:
Base 5: $5^{3n} \times 5^{2n^2+2n} = 5^{2n^2+5n}$
Base 3: $3^{3n} \times 3^{2-3n} = 3^2$
Expression: $\frac{5^{2n^2+5n} \times 3^2}{3^2} = 5^{2n^2+5n}$
Set equal to 125 (which is 5^3):
$5^{2n^2+5n} = 5^3 \implies 2n^2 + 5n - 3 = 0$
Factorise: $(2n-1)(n+3) = 0 \implies \mathbf{n = 0.5}$ or $\mathbf{n = -3}$.

You Do 3

Prime bases: $21^n = 3^n \times 7^n$.
Numerator:
Base 7: $7^n \times 7^{n^2-3n} = 7^{n^2-2n}$
Base 3: $3^n \times 3^{1-n} = 3^1$
Expression inside bracket: $\frac{7^{n^2-2n} \times 3^1}{3^1} = 7^{n^2-2n}$
Apply outer exponent (-1): 7^{-n^2+2n}
Set equal to $\frac{1}{343} = 7^{-3}$:
$-n^2 + 2n = -3 \implies n^2 - 2n - 3 = 0$
$(n-3)(n+1) = 0 \implies \mathbf{n = 3}$ or $\mathbf{n = -1}$.

Challenge 1

Express all terms using base 3:
$27^x = (3^3)^x = 3^{3x}, \quad 9^{2x} = (3^2)^{2x} = 3^{4x}, \quad 729 = 3^6$
Substitute into the equation:
$\frac{3^{3x} \times 3^{x^2}}{3^{4x}} = 3^6 \implies \frac{3^{x^2+3x}}{3^{4x}} = 3^6$
$3^{x^2+3x-4x} = 3^6 \implies 3^{x^2-x} = 3^6$
$x^2 - x - 6 = 0 \implies (x-3)(x+2) = 0 \implies \mathbf{x = 3}$ or $\mathbf{x = -2}$.

Challenge 2

Convert Equation 1 to base 2:

$2^x \times (2^2)^y = 2^9 \implies 2^{x+2y} = 2^9 \implies x + 2y = 9$ (Eq. A)

Convert Equation 2 to base 3:

$\frac{3^{x^2}}{(3^2)^y} = 3^3 \implies 3^{x^2-2y} = 3^3 \implies x^2 - 2y = 3$ (Eq. B)

Add Equation A and Equation B to eliminate y:

$(x + 2y) + (x^2 - 2y) = 9 + 3 \implies x^2 + x = 12 \implies x^2 + x - 12 = 0$

$(x + 4)(x - 3) = 0$. Since we are given $x > 0$, we must have $\mathbf{x = 3}$.

Substitute $x = 3$ into Eq. A: $3 + 2y = 9 \implies 2y = 6 \implies \mathbf{y = 3}$.

Extreme Challenge

Break down all composite bases into prime factors of 2 and 3:

$12^x = (2^2 \times 3)^x = 2^{2x} \times 3^x$

$18^{x^2} = (2 \times 3^2)^{x^2} = 2^{x^2} \times 3^{2x^2}$

$4^{x^2} = (2^2)^{x^2} = 2^{2x^2}$

Substitute these into the main expression:

Numerator: $(2^{2x} \times 3^x) \times (2^{x^2} \times 3^{2x^2}) \times 2^{4-2x}$

Combine the powers of 2 in the numerator: $2x + x^2 + 4 - 2x = x^2 + 4$

Combine the powers of 3 in the numerator: $x + 2x^2$

Numerator $= 2^{x^2+4} \times 3^{2x^2+x}$

Denominator: $3^{2x^2+x} \times 2^{2x^2}$

Fraction: $\frac{2^{x^2+4} \times 3^{2x^2+x}}{2^{2x^2} \times 3^{2x^2+x}}$

The base 3 terms cancel out completely.

For base 2: $2^{(x^2+4)-2x^2} = 2^{4-x^2}$

We are given the RHS is 0.125.

$0.125 = \frac{1}{8} = 2^{-3}$.

Therefore: $2^{4-x^2} = 2^{-3}$

$4 - x^2 = -3 \implies x^2 = 7 \implies \mathbf{x = \pm\sqrt{7}}$.