Completing the Square (Negative Quadratics)

Practice Worksheet (Problem Solving)

I Do

(i) Express $-3x^2 - 18x + 4$ in the form $a - b(x + c)^2$ where a, b and c are integers.

.....

(ii) Using your answer to part (i), solve the equation $-3x^2 - 18x + 4 = 0$
Show clear algebraic working.
Give your answer in the form $\frac{\pm\sqrt{m}-n}{p}$ where m, n and p are integers.

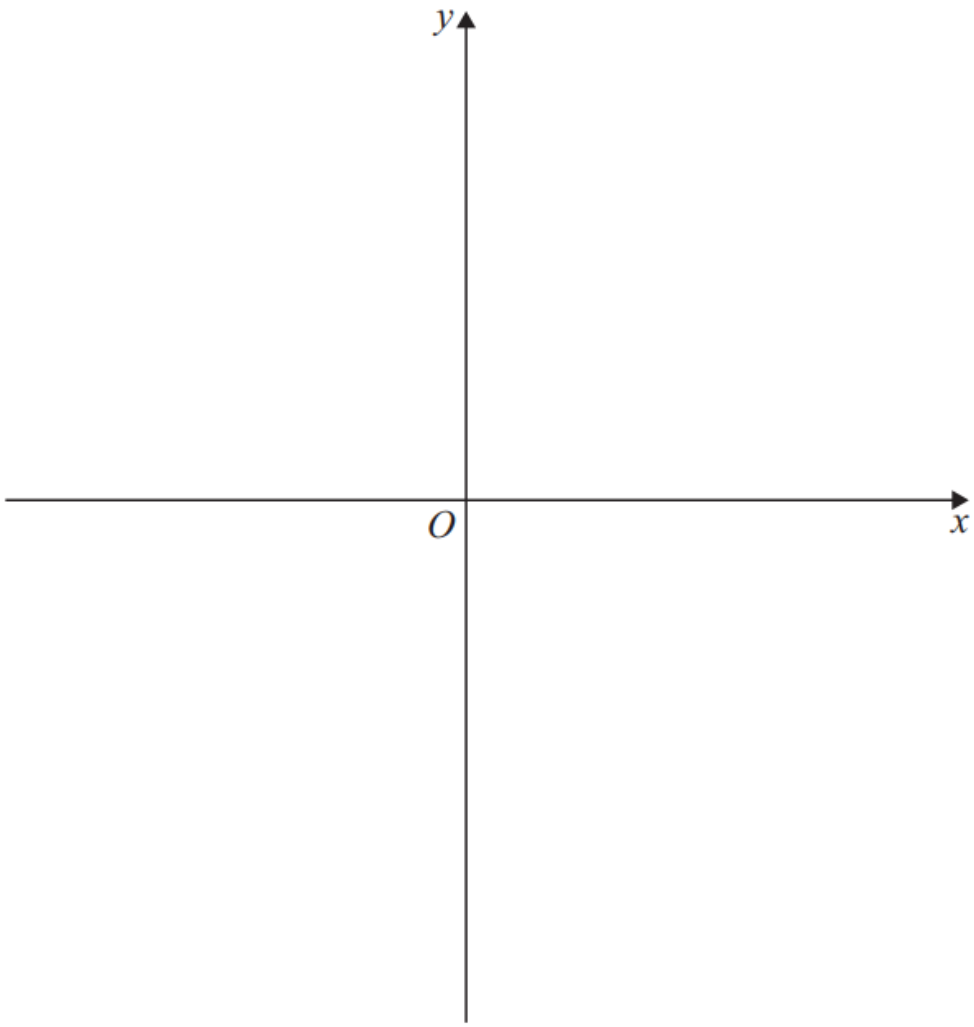
$x =$ .....

Continued on the next page: sketch the graph (3 marks).
Total for Question, including the sketch: 9 marks.

I Do — continued

(iii) Hence, sketch the graph of $\mathbf{y = -3x^2 - 18x + 4}$, clearly showing the coordinates of the turning point and the exact coordinates of any intercepts with the coordinate axes.

(3 marks)



Sketch not to scale. Label exact coordinates.
Total for Question is 9 marks.

We Do

(i) Express $-3x^2 - 24x + 5$ in the form $a - b(x + c)^2$ where a, b and c are integers.

.....

(ii) Using your answer to part (i), solve the equation $-3x^2 - 24x + 5 = 0$
Show clear algebraic working.
Give your answer in the form $\frac{\pm\sqrt{m}-n}{p}$ where m, n and p are integers.

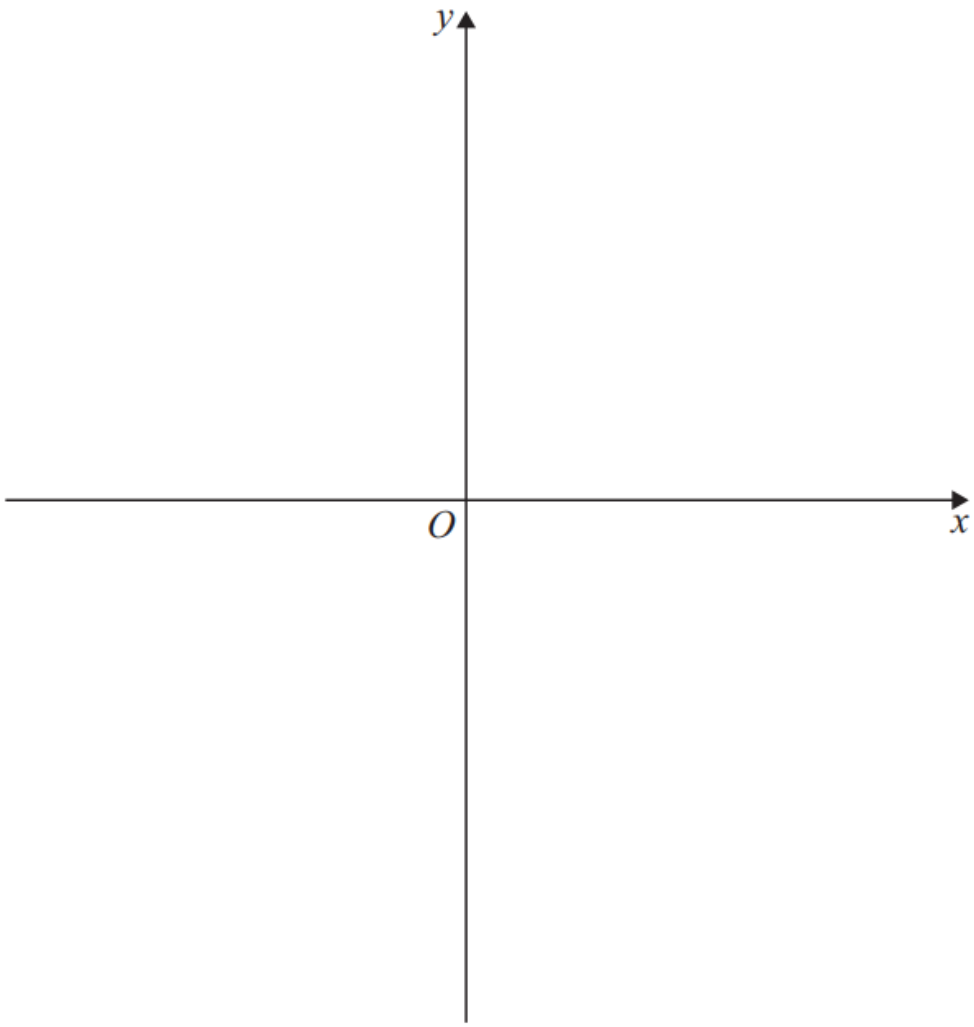
$x =$ .....

Continued on the next page: sketch the graph (3 marks).
Total for Question, including the sketch: 9 marks.

We Do — continued

(iii) Hence, sketch the graph of $\mathbf{y = -3x^2 - 24x + 5}$, clearly showing the coordinates of the turning point and the exact coordinates of any intercepts with the coordinate axes.

(3 marks)



Sketch not to scale. Label exact coordinates.
Total for Question is 9 marks.

You Do 1

(i) Express $-4x^2 - 40x - 11$ in the form $a - b(x + c)^2$ where a, b and c are integers.

.....

(ii) Using your answer to part (i), solve the equation $-4x^2 - 40x - 11 = 0$
Show clear algebraic working.
Give your answer in the form $\frac{\pm\sqrt{m}-n}{p}$ where m, n and p are integers.

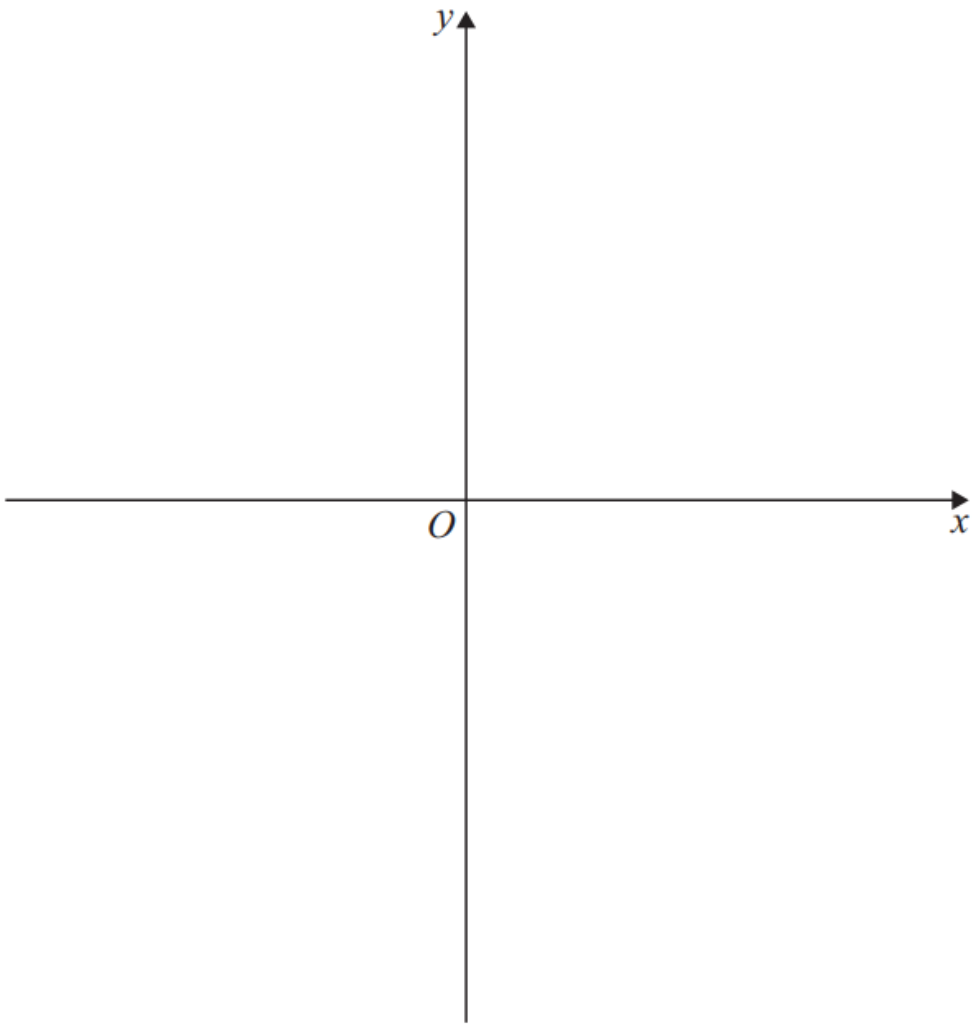
$x =$ .....

Continued on the next page: sketch the graph (3 marks).
Total for Question, including the sketch: 9 marks.

You Do 1 — continued

(iii) Hence, sketch the graph of $\mathbf{y = -4x^2 - 40x - 11}$, clearly showing the coordinates of the turning point and the exact coordinates of any intercepts with the coordinate axes.

(3 marks)



Sketch not to scale. Label exact coordinates.
Total for Question is 9 marks.

You Do 2

(i) Express $15 - 8x - 2x^2$ in the form $a - b(x + c)^2$ where a, b and c are integers.

.....

(ii) Using your answer to part (i), solve the equation $15 - 8x - 2x^2 = 0$
Show clear algebraic working.
Give your answer in the form $\frac{\pm\sqrt{m}-n}{p}$ where m, n and p are integers.

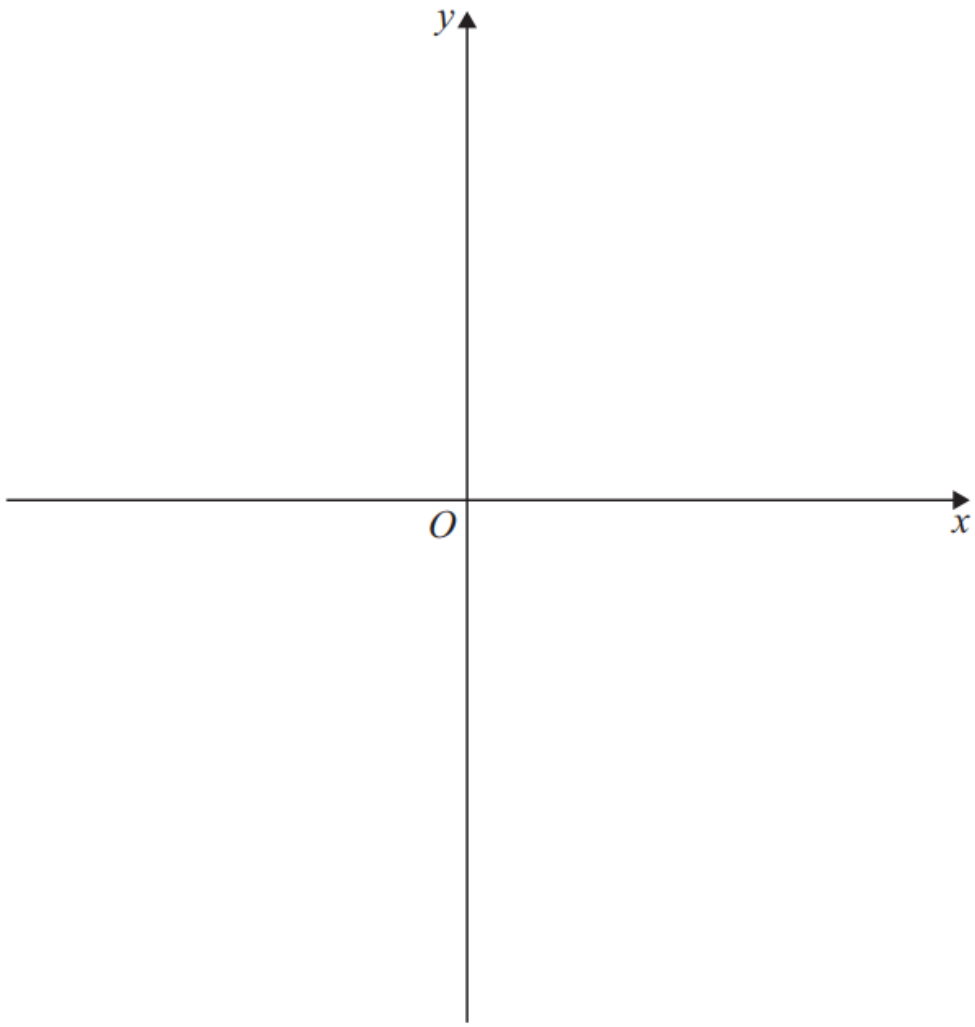
$x =$ .....

Continued on the next page: sketch the graph (3 marks).
Total for Question, including the sketch: 9 marks.

You Do 2 — continued

(iii) Hence, sketch the graph of $\mathbf{y = -2x^2 - 8x + 15}$, clearly showing the coordinates of the turning point and the exact coordinates of any intercepts with the coordinate axes.

(3 marks)



Sketch not to scale. Label exact coordinates.
Total for Question is 9 marks.

You Do 3

(i) Express $-5x^2 - 20x + 7$ in the form $a - b(x + c)^2$ where a, b and c are integers.

.....

(ii) Using your answer to part (i), solve the equation $-5x^2 - 20x + 7 = 0$
Show clear algebraic working.
Give your answer in the form $\frac{\pm\sqrt{m}-n}{p}$ where m, n and p are integers.

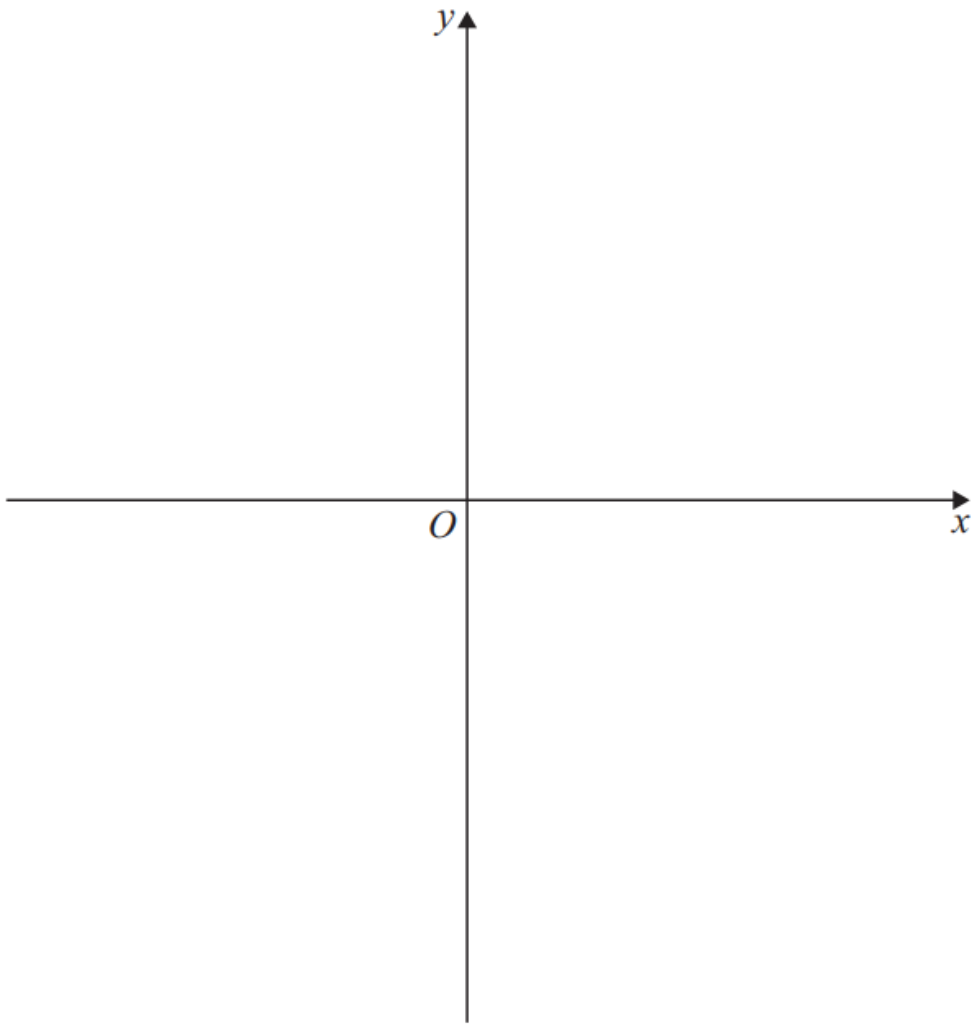
$x =$ .....

Continued on the next page: sketch the graph (3 marks).
Total for Question, including the sketch: 9 marks.

You Do 3 — continued

(iii) Hence, sketch the graph of $\mathbf{y = -5x^2 - 20x + 7}$, clearly showing the coordinates of the turning point and the exact coordinates of any intercepts with the coordinate axes.

(3 marks)



Sketch not to scale. Label exact coordinates.
Total for Question is 9 marks.

Challenge 1

(i) Express $5 - 6x - 2x^2$ in the form $a - b(x + c)^2$ where a, b and c are rational numbers.

.....

(ii) Hence, solve the equation $5 - 6x - 2x^2 = 0$
Show clear algebraic working.
Give your answer in the form $\frac{\pm\sqrt{m}-n}{p}$ where m, n and p are integers.

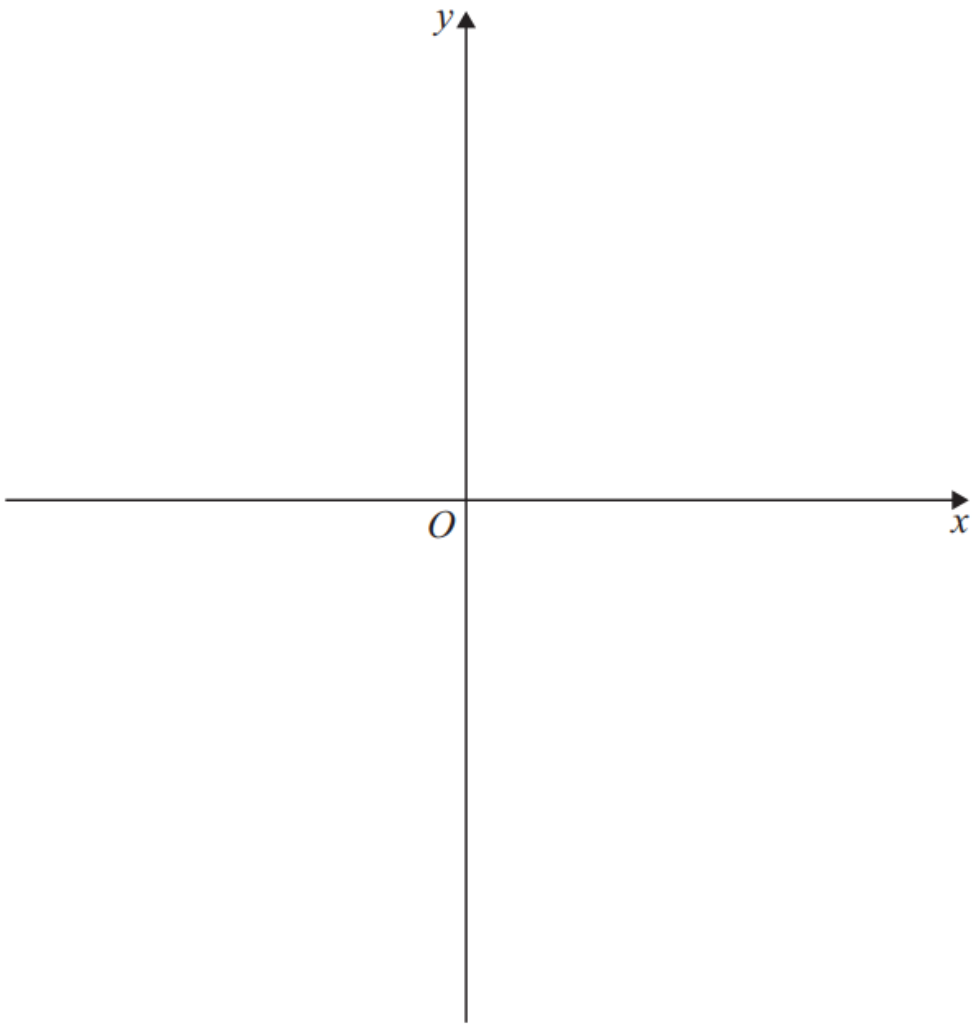
$x =$ .....

Continued on the next page: sketch the graph (3 marks).
Total for Question, including the sketch: 10 marks.

Challenge 1 — continued

(iii) Hence, sketch the graph of $\mathbf{y = -2x^2 - 6x + 5}$, clearly showing the coordinates of the turning point and the exact coordinates of any intercepts with the coordinate axes.

(3 marks)



Sketch not to scale. Label exact coordinates.
Total for Question is 10 marks.

Challenge 2

(i) The curve C has the equation $y = 8 - 12x - 3x^2$.
By completing the square, find the coordinates of its turning point.

(ii) State whether this turning point is a maximum or a minimum.

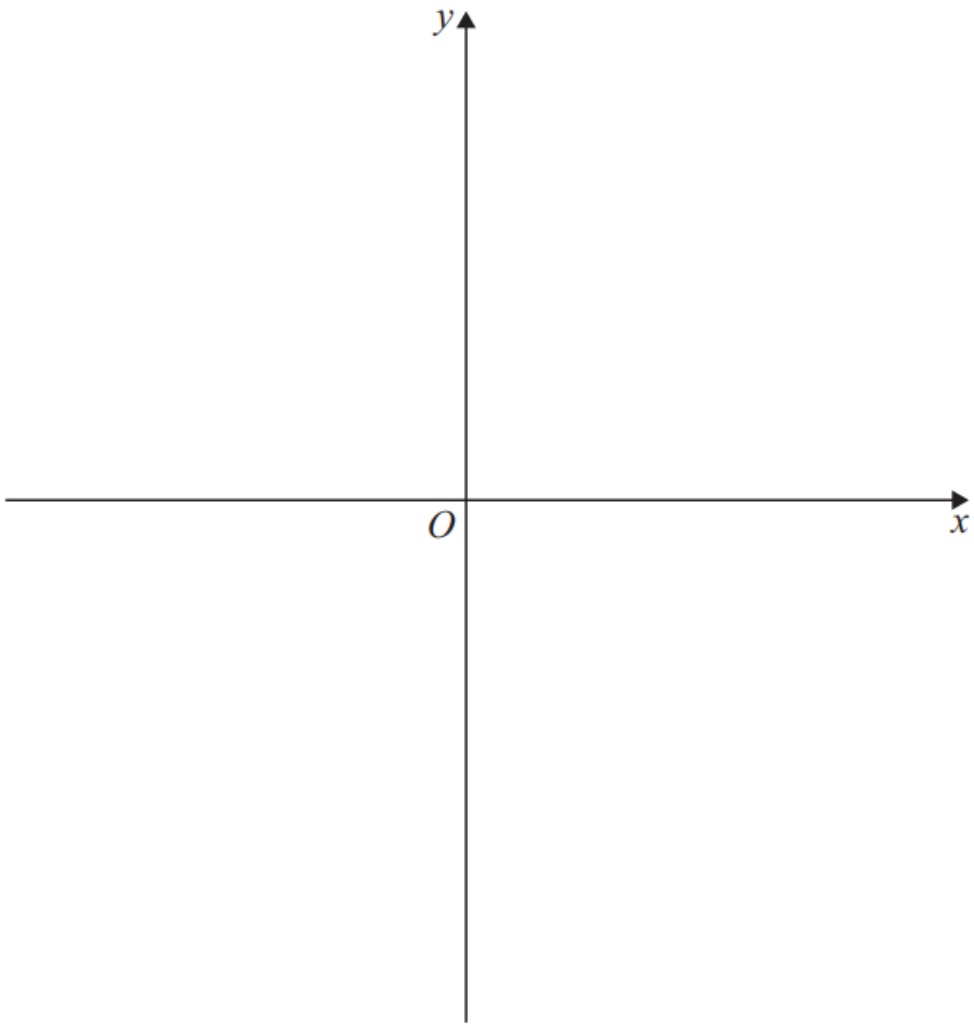
Turning Point: (. . . . . , ...)

Continued on the next page: sketch the graph (3 marks).
Total for Question, including the sketch: 7 marks.

Challenge 2 — continued

(iii) Hence, sketch the graph of $\mathbf{y = -3x^2 - 12x + 8}$, clearly showing the coordinates of the turning point and the exact coordinates of any intercepts with the coordinate axes.

(3 marks)



Sketch not to scale. Label exact coordinates.
Total for Question is 7 marks.

Extreme Challenge

A rectangle has length $(10 - x)$ cm and width $(3x + 2)$ cm.

(a) Show that the area of the rectangle, $A\,\text{cm}^2$, is given by $A = 20 + 28x - 3x^2$.

(b) By completing the square, find the maximum possible area of this rectangle, and the exact value of x for which this maximum area occurs.

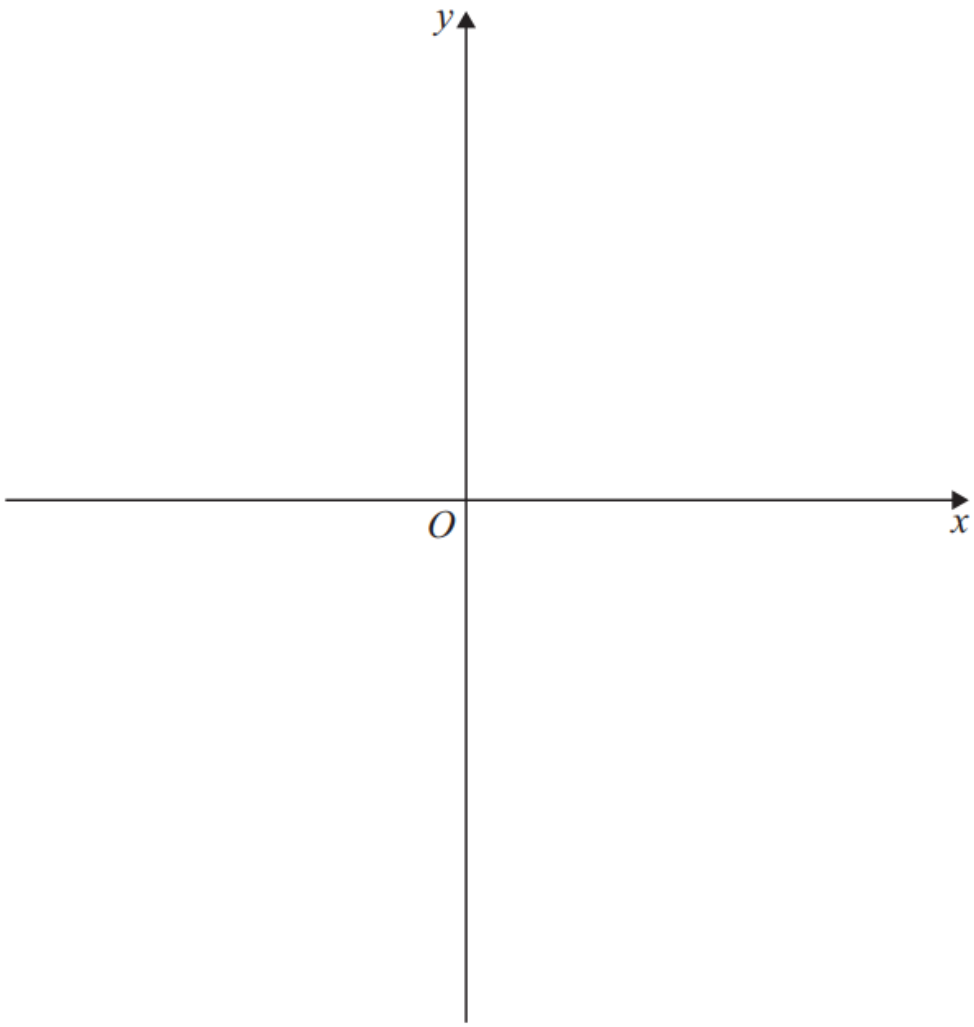
Max Area = ... cm^2 at $x =$ .....

Continued on the next page: sketch the graph (3 marks).
Total for Question, including the sketch: 9 marks.

Extreme Challenge — continued

(c) Hence, sketch the graph of $\mathbf{y = -3x^2 + 28x + 20}$, clearly showing the coordinates of the turning point and the exact coordinates of any intercepts with the coordinate axes.

(3 marks)



Sketch not to scale. Label exact coordinates.
Total for Question is 9 marks.

Worked Answers

I Do

(i) Factorise -3 out of the x terms: $-3(x^2 + 6x) + 4$
Complete the square inside the bracket: $-3[(x + 3)^2 - 9] + 4$
Expand the outer bracket: $-3(x + 3)^2 + 27 + 4$
Simplify: $\mathbf{31 - 3(x + 3)^2}$
(ii) Set equal to zero: $31 - 3(x + 3)^2 = 0$
$3(x + 3)^2 = 31 \implies (x + 3)^2 = \frac{31}{3}$
Multiply top and bottom by 3 to make the denominator a square number: $(x + 3)^2 = \frac{93}{9}$
Square root both sides: $x + 3 = \pm\frac{\sqrt{93}}{3}$
Subtract 3 (which is $\frac{9}{3}$): $x = \pm\frac{\sqrt{93}}{3} - \frac{9}{3}$
$x = \frac{\pm\sqrt{\mathbf{93}}\mathbf{-9}}{\mathbf{3}}$

We Do

(i) Factorise -3: $-3(x^2 + 8x) + 5$
Complete the square: $-3[(x + 4)^2 - 16] + 5$
Expand: $-3(x + 4)^2 + 48 + 5 = \mathbf{53 - 3(x + 4)^2}$
(ii) Solve: $53 - 3(x + 4)^2 = 0 \implies 3(x + 4)^2 = 53$
$(x + 4)^2 = \frac{53}{3} \implies (x + 4)^2 = \frac{159}{9}$
$x + 4 = \pm\frac{\sqrt{159}}{3}$
$x = \pm\frac{\sqrt{159}}{3} - \frac{12}{3} = \frac{\pm\sqrt{\mathbf{159}}\mathbf{-12}}{\mathbf{3}}$

You Do 1

(i) $-4(x^2 + 10x) - 11 = -4[(x + 5)^2 - 25] - 11 = -4(x + 5)^2 + 100 - 11 = \mathbf{89 - 4(x + 5)^2}$
(ii) $89 - 4(x + 5)^2 = 0 \implies 4(x + 5)^2 = 89 \implies (x + 5)^2 = \frac{89}{4}$
$x + 5 = \pm\frac{\sqrt{89}}{2}$
$x = \frac{\pm\sqrt{\mathbf{89}}\mathbf{-10}}{\mathbf{2}}$

You Do 2

(i) $-2(x^2+4x)+15 = -2[(x+2)^2-4]+15 = -2(x+2)^2+8+15 = \mathbf{23-2(x+2)^2}$

(ii) $23-2(x+2)^2=0 \implies 2(x+2)^2=23 \implies (x+2)^2=\frac{23}{2}=\frac{46}{4}$

$x+2=\pm\frac{\sqrt{46}}{2}$

$x=\frac{\pm\sqrt{\mathbf{46}}-\mathbf{4}}{\mathbf{2}}$

You Do 3

(i) $-5(x^2+4x)+7 = -5[(x+2)^2-4]+7 = -5(x+2)^2+20+7 = \mathbf{27-5(x+2)^2}$

(ii) $27-5(x+2)^2=0 \implies 5(x+2)^2=27 \implies (x+2)^2=\frac{27}{5}=\frac{135}{25}$

$x+2=\pm\frac{\sqrt{135}}{5}$

$x=\frac{\pm\sqrt{\mathbf{135}}-\mathbf{10}}{\mathbf{5}}$

Challenge 1

(i) $-2(x^2+3x)+5 = -2\left[\left(x+\frac{3}{2}\right)^2-\frac{9}{4}\right]+5 = -2\left(x+\frac{3}{2}\right)^2+\frac{18}{4}+5$

$\frac{18}{4}+5=\frac{9}{2}+\frac{10}{2}=\frac{19}{2}$

Form: $\frac{\mathbf{19}}{\mathbf{2}}-\mathbf{2}\left(\mathbf{x}+\frac{\mathbf{3}}{\mathbf{2}}\right)^{\mathbf{2}}$ (or $9.5-2(x+1.5)^2$)

(ii) $2\left(x+\frac{3}{2}\right)^2=\frac{19}{2} \implies \left(x+\frac{3}{2}\right)^2=\frac{19}{4}$

$x+\frac{3}{2}=\pm\frac{\sqrt{19}}{2}$

$x=\frac{\pm\sqrt{\mathbf{19}}-\mathbf{3}}{\mathbf{2}}$

Challenge 2

$y=-3(x^2+4x)+8=-3[(x+2)^2-4]+8=20-3(x+2)^2$.

The maximum value of y is 20, which occurs when the squared bracket is zero (i.e. when $x+2=0 \implies x=-2$).

Turning point: $(\mathbf{-2, 20})$. It is a **maximum** (due to the negative x^2 coefficient).

Extreme Challenge

(a) Area $=(10-x)(3x+2)=30x+20-3x^2-2x=\mathbf{20+28x-3x^2}$.

(b) Complete the square: $-3\left(x^2-\frac{28}{3}x\right)+20=-3\left[\left(x-\frac{14}{3}\right)^2-\frac{196}{9}\right]+20$

$=-3\left(x-\frac{14}{3}\right)^2+\frac{196}{3}+\frac{60}{3}=\frac{256}{3}-3\left(x-\frac{14}{3}\right)^2$

The maximum area is $\frac{\mathbf{256}}{\mathbf{3}}$ $\mathbf{cm^2}$, and this occurs exactly when $\mathbf{x}=\frac{\mathbf{14}}{\mathbf{3}}$.

I Do — sketch answer

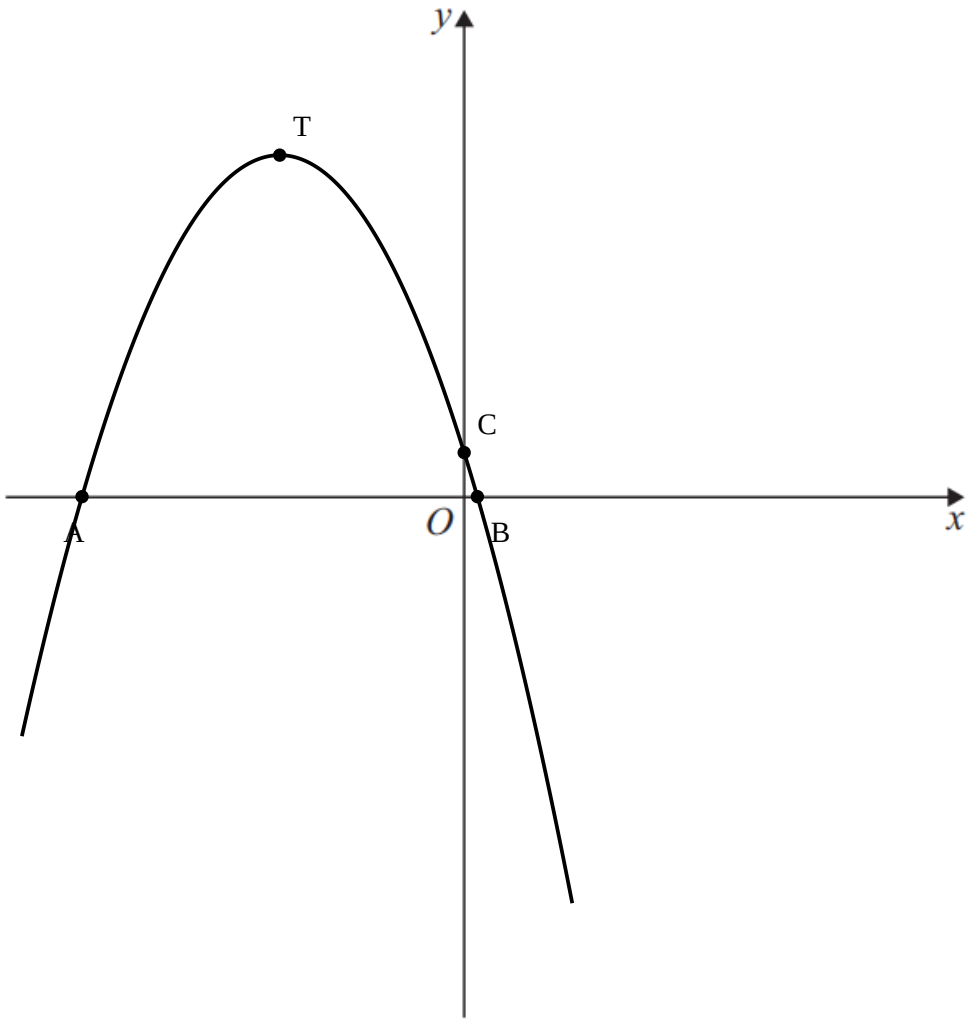
(iii) $y = -3x^2 - 18x + 4$

T: Turning point $(-3, 31)$, a maximum.
A: x-intercept $((-9 - \sqrt{93})/3, 0)$.
B: x-intercept $((-9 + \sqrt{93})/3, 0)$.
C: y-intercept $(0, 4)$.

The curve opens downwards and is symmetric about $x = -3$. Set $y = 0$ for A and B; set $x = 0$ for C.

3 marks: 1 for the downward-opening shape and correct maximum; 1 for both exact x-intercepts; 1 for the correct y-intercept.

Sketch not to scale; point labels refer to the exact coordinates above.



We Do — sketch answer

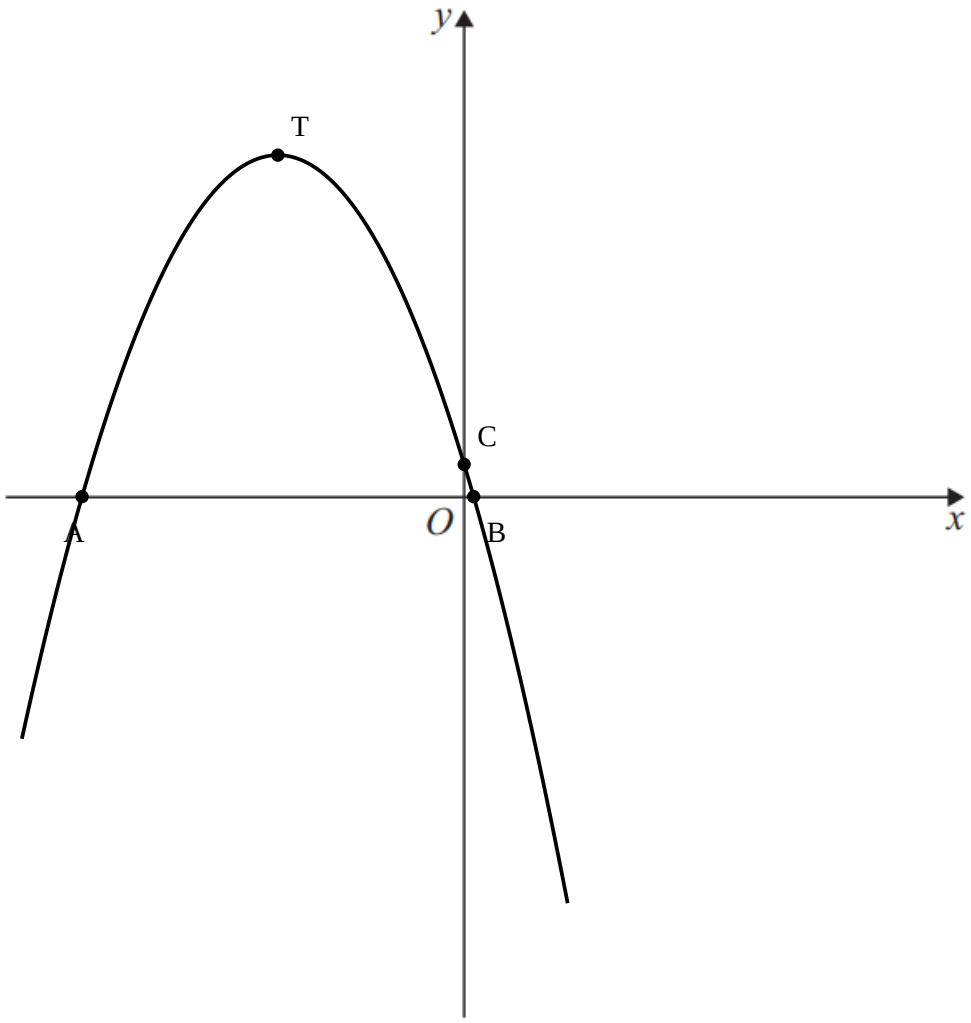
(iii) $y = -3x^2 - 24x + 5$

T: Turning point $(-4, 53)$, a maximum.
A: x-intercept $((-12 - \sqrt{159})/3, 0)$.
B: x-intercept $((-12 + \sqrt{159})/3, 0)$.
C: y-intercept $(0, 5)$.

The curve opens downwards and is symmetric about $x = -4$. Set $y = 0$ for A and B; set $x = 0$ for C.

3 marks: 1 for the downward-opening shape and correct maximum; 1 for both exact x-intercepts; 1 for the correct y-intercept.

Sketch not to scale; point labels refer to the exact coordinates above.



You Do 1 — sketch answer

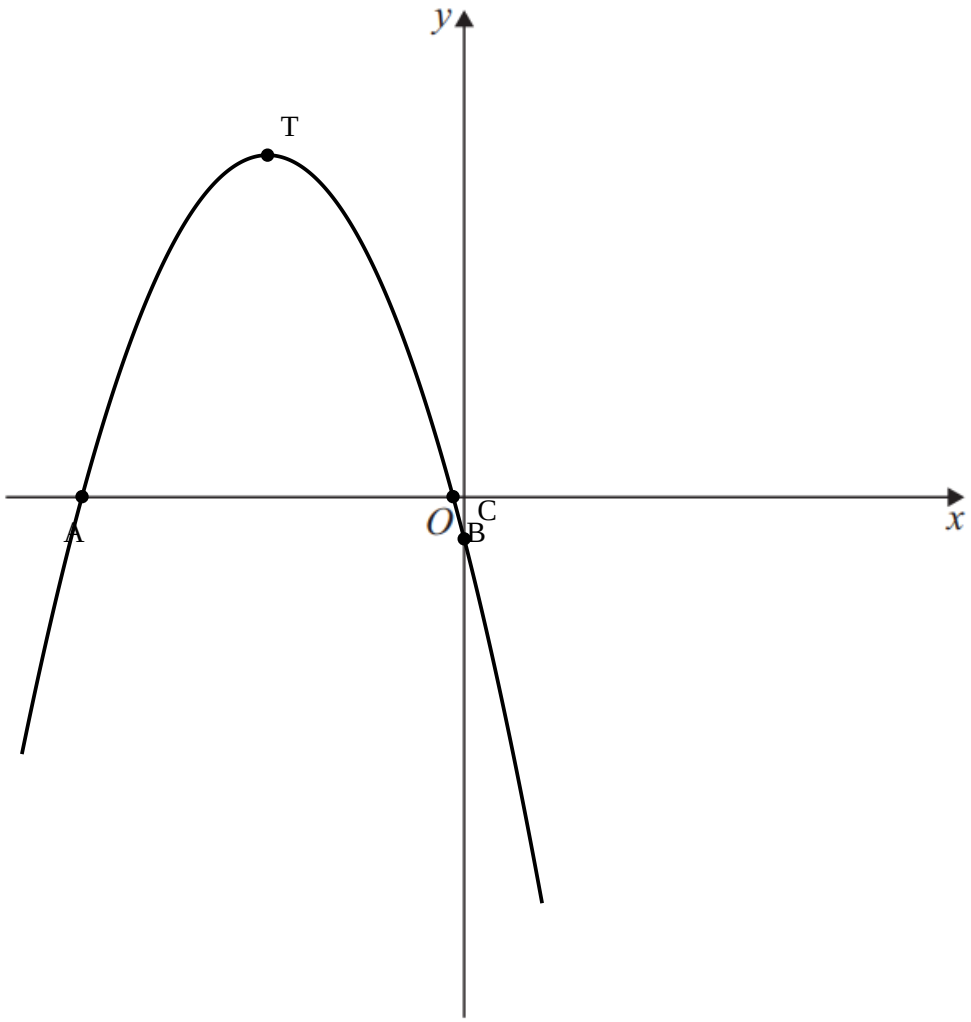
(iii) $y = -4x^2 - 40x - 11$

T: Turning point $(-5, 89)$, a maximum.
A: x-intercept $((-10 - \sqrt{89})/2, 0)$.
B: x-intercept $((-10 + \sqrt{89})/2, 0)$.
C: y-intercept $(0, -11)$.

The curve opens downwards and is symmetric about $x = -5$. Set $y = 0$ for A and B; set $x = 0$ for C.

3 marks: 1 for the downward-opening shape and correct maximum; 1 for both exact x-intercepts; 1 for the correct y-intercept.

Sketch not to scale; point labels refer to the exact coordinates above.



You Do 2 — sketch answer

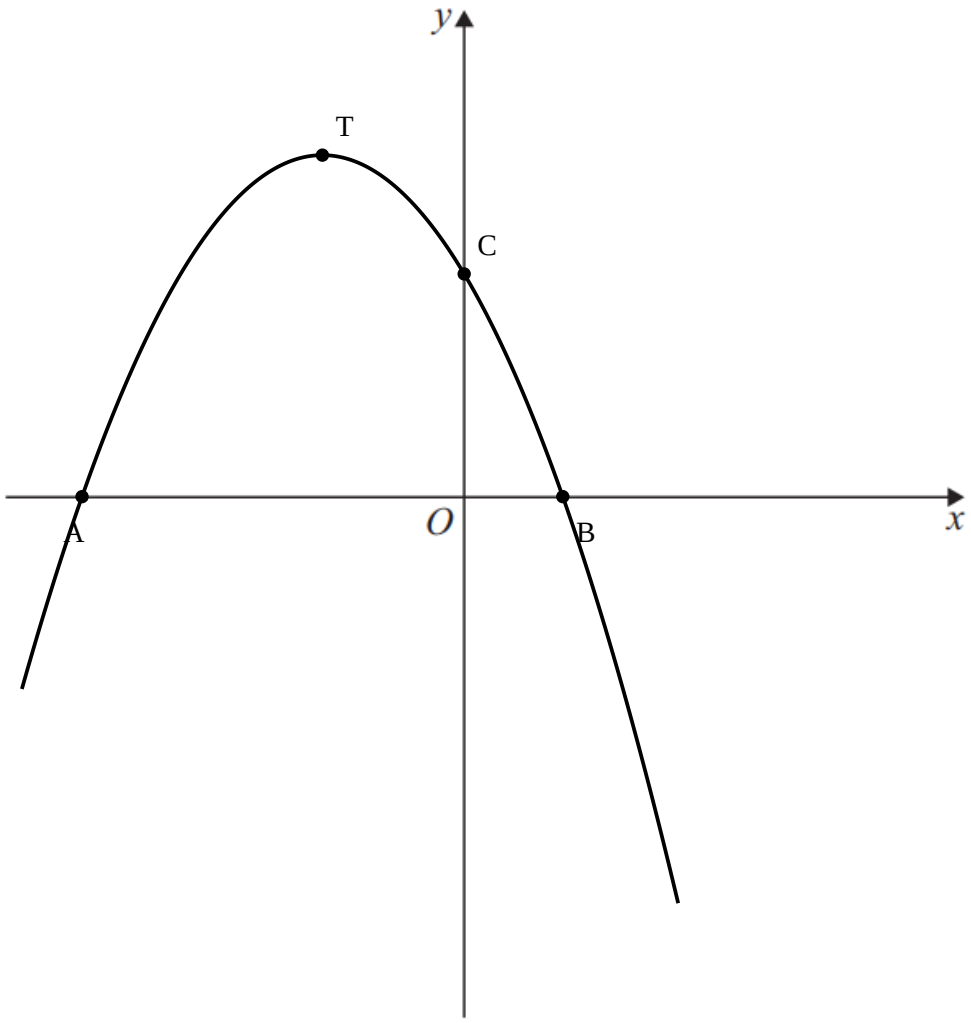
(iii) $y = -2x^2 - 8x + 15$

T: Turning point $(-2, 23)$, a maximum.
A: x-intercept $((-4 - \sqrt{46})/2, 0)$.
B: x-intercept $((-4 + \sqrt{46})/2, 0)$.
C: y-intercept $(0, 15)$.

The curve opens downwards and is symmetric about $x = -2$. Set $y = 0$ for A and B; set $x = 0$ for C.

3 marks: 1 for the downward-opening shape and correct maximum; 1 for both exact x-intercepts; 1 for the correct y-intercept.

Sketch not to scale; point labels refer to the exact coordinates above.



You Do 3 — sketch answer

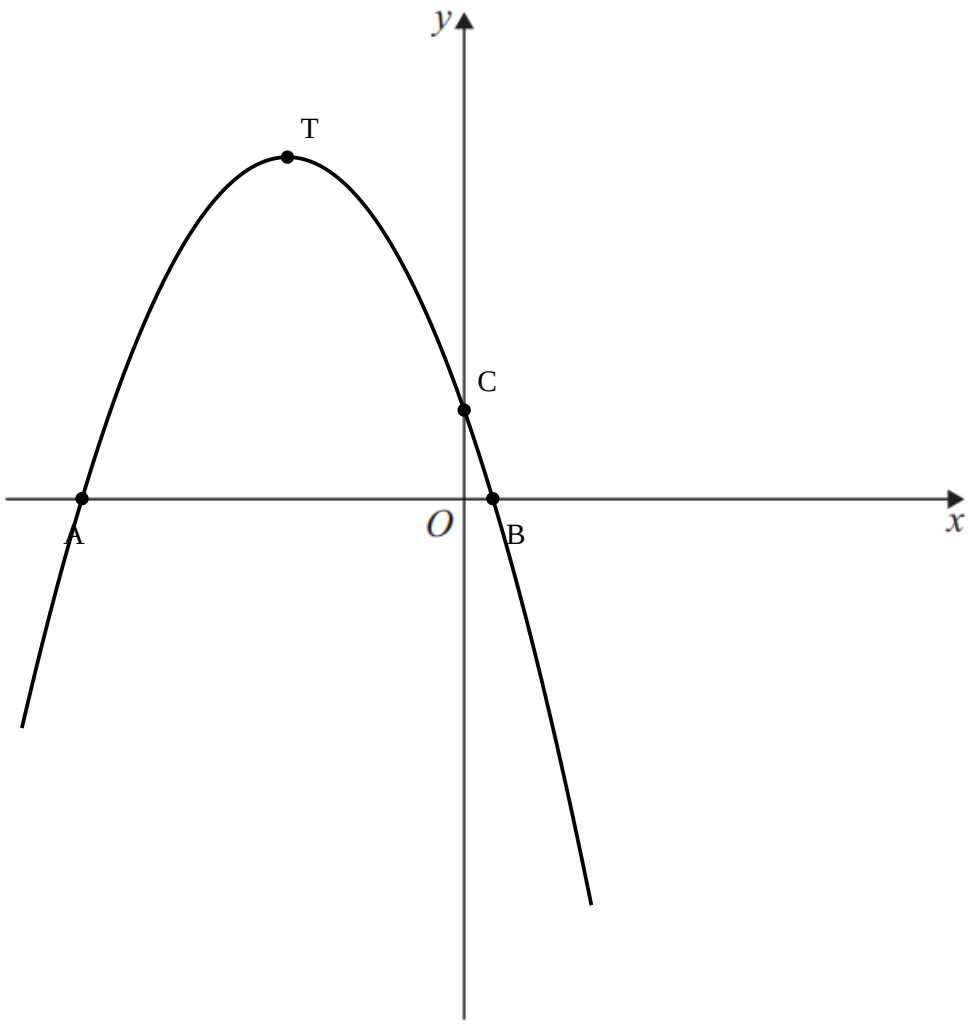
(iii) $y = -5x^2 - 20x + 7$

T: Turning point $(-2, 27)$, a maximum.
A: x-intercept $((-10 - \sqrt{135})/5, 0)$.
B: x-intercept $((-10 + \sqrt{135})/5, 0)$.
C: y-intercept $(0, 7)$.

The curve opens downwards and is symmetric about $x = -2$. Set $y = 0$ for A and B; set $x = 0$ for C.

3 marks: 1 for the downward-opening shape and correct maximum; 1 for both exact x-intercepts; 1 for the correct y-intercept.

Sketch not to scale; point labels refer to the exact coordinates above.



Challenge 1 — sketch answer

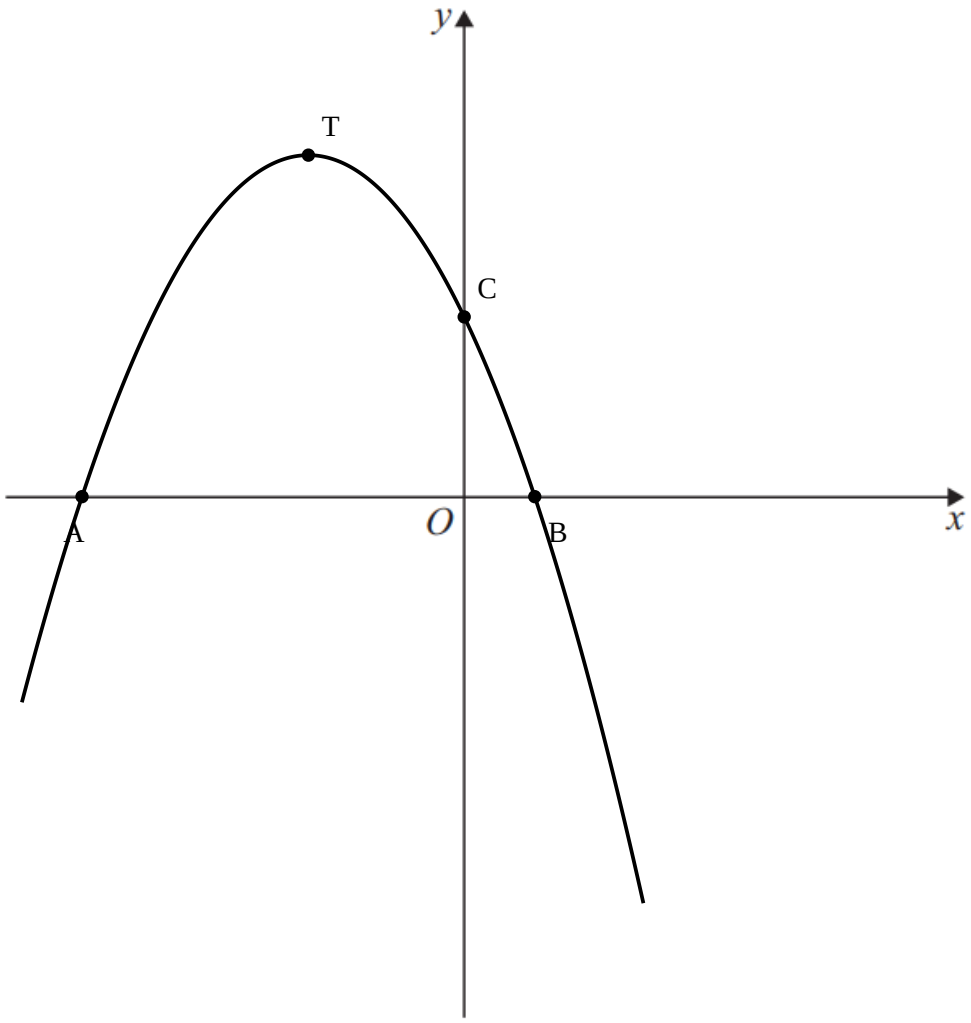
(iii) $y = -2x^2 - 6x + 5$

T: Turning point $(-3/2, 19/2)$, a maximum.
A: x-intercept $((-3 - \sqrt{19})/2, 0)$.
B: x-intercept $((-3 + \sqrt{19})/2, 0)$.
C: y-intercept $(0, 5)$.

The curve opens downwards and is symmetric about $x = -3/2$. Set $y = 0$ for A and B; set $x = 0$ for C.

3 marks: 1 for the downward-opening shape and correct maximum; 1 for both exact x-intercepts; 1 for the correct y-intercept.

Sketch not to scale; point labels refer to the exact coordinates above.



Challenge 2 — sketch answer

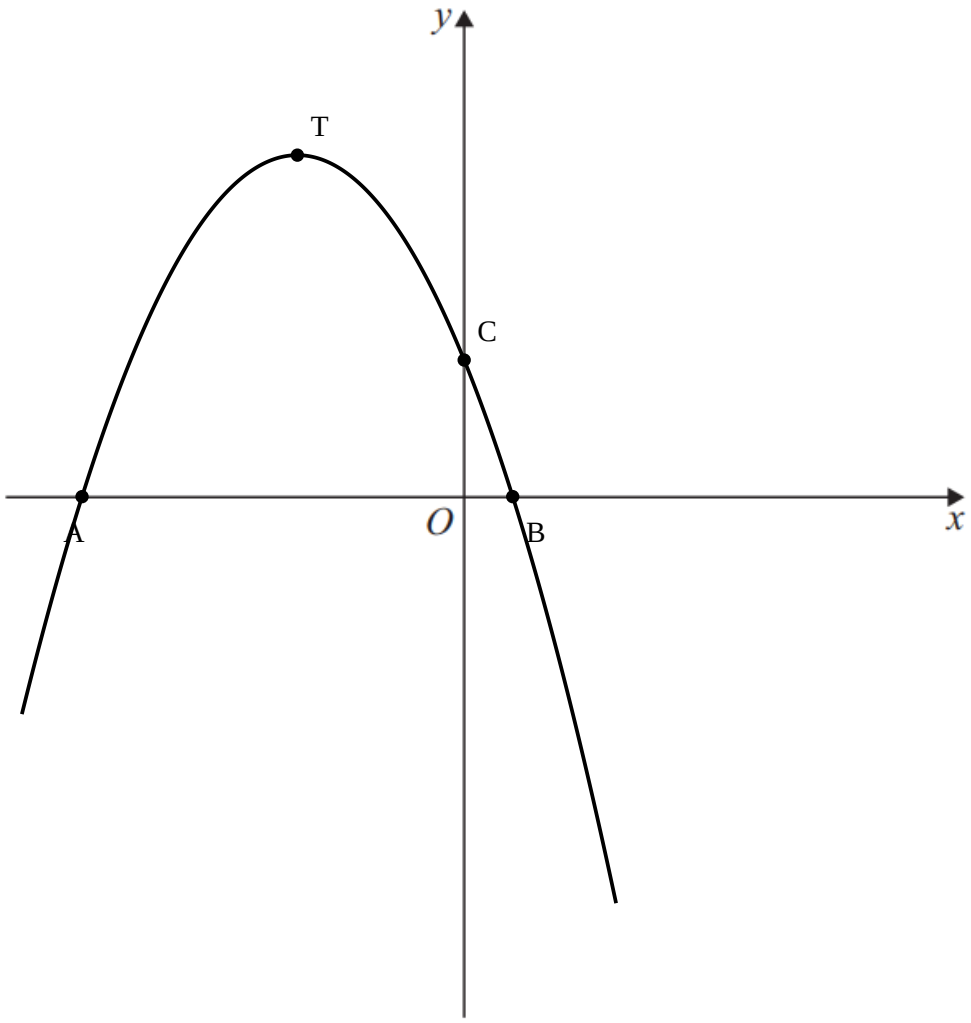
(iii) $y = -3x^2 - 12x + 8$

T: Turning point $(-2, 20)$, a maximum.
A: x-intercept $((-6 - 2\sqrt{15})/3, 0)$.
B: x-intercept $((-6 + 2\sqrt{15})/3, 0)$.
C: y-intercept $(0, 8)$.

The curve opens downwards and is symmetric about $x = -2$. Set $y = 0$ for A and B; set $x = 0$ for C.

3 marks: 1 for the downward-opening shape and correct maximum; 1 for both exact x-intercepts; 1 for the correct y-intercept.

Sketch not to scale; point labels refer to the exact coordinates above.



Extreme Challenge — sketch answer

(c) $y = -3x^2 + 28x + 20$

T: Turning point (14/3, 256/3), a maximum.
A: x-intercept $(-2/3, 0)$.
B: x-intercept (10, 0).
C: y-intercept (0, 20).

The curve opens downwards and is symmetric about $x = 14/3$. Set $y = 0$ for A and B; set $x = 0$ for C.

3 marks: 1 for the downward-opening shape and correct maximum; 1 for both exact x-intercepts; 1 for the correct y-intercept.

Sketch the full quadratic. For the original rectangle, positive side lengths require $-2/3 < x < 10$.

Sketch not to scale; point labels refer to the exact coordinates above.

